# Bitcoin-based triangular arbitrage with the Euro/U.S. dollar as a foreign futures hedge: forecasting with a bivariate GARCH model

**Abstract**

People buy bitcoin for one of two reasons: to use it as a currency or to use it as an investment asset. However, because of the wildly fluctuating bitcoin price and the high volatility of returns, bitcoin has failed to play either of these roles well. In this environment, bitcoin-based currency exchange strategies are proposed. The most sophisticated strategy, defined as bitcoin-based triangular arbitrage, combines foreign exchanges in the bitcoin market and reverse foreign exchange spot transactions. To reduce exposure to risk, an FX futures contract is used as a hedging instrument. This paper applies a bivariate DCC-GARCH model with multivariate Gaussian-distributed disturbances to measure the joint density of the returns of the underlying asset and its hedge. For each asset, Student's *t* density is suggested for the observed leptokurtosis and fat tails. Based on a time-dependent variance-covariance matrix, a dynamic optimal hedge ratio is formed, with a time-varying correlation series as a by-product. Empirical results are obtained using Euros and U.S. dollars over the period from 21 April 2014 to 21 September 2018. Multiple rolling one-step-ahead forecasts are generated. Finally, a comparison of direct bitcoin investing and our proposed strategies is presented.

Key words: bitcoin, bitcoin exchange rate, triangular arbitrage, optimal hedge ratio, and DCC-GARCH model

## 1 Introduction

Bitcoin, the first widely used cryptocurrency, was devised with the intent of becoming a virtual currency free of any financial authority to facilitate payments from peer to peer in its network (Nakamoto, 2008). The most striking feature of bitcoin that differentiates it from e-commerce payment is its decentralized architecture, which is seen by many as an innovation in the established money system (Böhme et al., 2015). However, it was soon discovered that those dealing in digital currencies were primarily interested in an alternative investment rather than an alternative transaction system (Glaser et al., 2014). Bitcoin as a unique investment asset has experienced rapid growth and occupies first market place among all cryptocurrencies, partially due to its first-mover advantage (Luther, 2016). Data from Blockchain.info show that the major market capitalization of bitcoin in USD increased from $0.26 million in August 2010 to $114.08 billion in September 2018, peaking at 323.07 billion on 17 December 2017.

However, bitcoin's two major roles, as a currency and as an investment asset, have been questioned by a number of researchers. As a currency, bitcoin needs to perform the three functions of money: It must serve as a unit of account, a medium of exchange, and a store of value. Yermack (2013) and Ali et al. (2014) posit that bitcoin performs poorly in all three of these functions. As a financial asset, bitcoin is expected to bring a positive risk-adjusted return by investors who employ a buy-and-hold strategy. Without any intrinsic value, bitcoin is said to carry a zero fundamental price and to have a substantial speculative bubble component in its price (Cheah & Fry, 2015). Dong & Dong (2015) point out that bitcoin investing has characteristics such as a liquidity discount, an unsystematic risk premium, and a low return that are associated with higher risk.

Though several factors contribute to its immaturity as a currency and its high risk as an investment, bitcoin's wildly fluctuating prices and excessive volatility appear to have substantial influence. During our period of observation, from 21 April 2014 to 21 September 2018, the bitcoin price index in U.S. dollars ranged from $183.07 to $18,674.48; its price index in Euros varied between €156.97 and €15,528.90 (see Figure 1). Such price variation creates multiple problems. First, the high price of a bitcoin makes it extremely inconvenient when people quote the price of a good or service in bitcoins due to all the decimal places in the price; the high price also limits the liquidity of bitcoin. Second, continual changes in the bitcoin price necessitate the frequent updating of bitcoin quotations for a good or service. Third, the wide range of bitcoin prices indicates a large risk in holding bitcoins. Previous studies have also found that bitcoin shows both very high volatility and potentially high returns, as well as a low correlation with other financial assets (Briere et al. , 2013; Yermack, 2013). Excessive volatility means excessive risk for investors and a consequent increase in the cost of managing risk. Low correlation with other assets implies ineffective hedging to manage the risk of the portfolio.

Bitcoin price is said to follow the law of supply and demand. However, because bitcoin supply is set by a rigid exogenous algorithm, bitcoin demand is the only market force determining the equilibrium price (Buchholz et al., 2012; Kristoufek, 2013; van Wijk, 2013). On the demand side, bitcoin's attractiveness as a medium of exchange, a store of value, and a favorable investment tool—or even fondness for its niche—is critical to determining its price. The alternation of positive and negative news has generated highly volatile price cycles (Ciaian et al., 2016).

This paper proposes several bitcoin-based foreign exchange strategies. In perhaps the simplest example, a speculator, on 22 April 2014, used 100 Euros to buy 0.2755 units of a bitcoin according to the previous day's closing bitcoin price in EUR. The individual then immediately sells the bitcoins at the previous day's closing price in USD for 138.6261 U.S. dollars. (Bitcoin prices on 21 April 2014 were retrieved from Bloomberg: USD/BTC =

503.18 and EUR/BTC = 363.00, where BTC denotes the bitcoin.) After this two-step transaction, 100 Euros has been exchanged for 138.6261 Euros at the rate implied by 138.6261/100 = 1.3863 USD/EUR. This cross rate between U.S. dollars and Euros, formed in the bitcoin market using bitcoin as the base currency, is defined as the bitcoin exchange rate of USD/EUR (Nan & Kaizoji, 2018a).

The bitcoin exchange rate provides a feasible way of exchanging fiat currency in the bitcoin market. Bitcoin's low transaction cost, efficiency and freedom are attractive features for this strategy. Nakamoto (2008), a pseudonym used by the individual or group claiming to have created bitcoin, wrote in a white paper that having a central authority working as a trusted third party "increases the transaction cost, limiting the minimum practical transaction size and cutting off possible small casual transactions." In the bitcoin system, trust is replaced by cryptographic proof to keep transactions decentralized from any authority so that bitcoin trading can be conducted from peer to peer with greater efficiency and lower cost. Moreover, worldwide 24-7 bitcoin transactions provide both abundant liquidity and a broad variety of fiat currencies for currency exchanges.

Pichl & Kaizoji (2017) studied the arbitrage opportunities in Bitcoin prices in different currencies. In this paper we investigate the Bitcoin arbitrage opportunities in detail. We can expand the simple example to illustrate a triangular arbitrage with the FX spot market. The closed FX exchange spot rate (the spot rate) on 22 April 2014 was 1.3793. The one-day spread between the two exchange rates was 0.0070, which takes account of 0.51% of the spot rate. Suppose now that the user brings his/her 138.6261 U.S. dollars to the FX spot market and trades for Euros at the price of 1.3793 (USD/EUR). This reverse operation in the FX spot market produces 100.5048 Euros. The one-round return based on this triangular arbitrage is approximately 0.50%. Dong & Dong (2015) found that there exists an opportunity for arbitrage by "spending bitcoin as currency" and that "arbitrage stickiness is persistent over time” potentially due to people’s preference for a buy-and-hold strategy with bitcoins. Nan & Kaizoji (2018a) found that the bitcoin exchange rate of USD/EUR is in long-run equilibrium with the spot rate of USD/EUR, with a zero risk premium term and a unity slope term that indicates unbiasedness. As a result, the return of this triangular arbitrage tends to be a martingale process in the long term; in the short term, deviation from equilibrium frequently occurs. However, due to the existence of an error correction mechanism, the discrepancy fluctuates around a zero mean. Given these results, we posit that the bitcoin market, in the long run, complies with the law of one price regarding the FX spot market in light of the market efficiency theory, but in the short run, an arbitrage opportunity may exist. The arbitrage paradox states that the market is efficient, yet a short-run arbitrage opportunity is simultaneously created when investors may not have sufficient incentives to observe the market (Grossman & Stiglitz, 1980; Akram et al, 2008).

It is possible that the risk of performing this triangular arbitrage can be reduced by using a hedging instrument. In fact, the FX futures contract may well serve this purpose. Nan & Kaizoji (2018b) conducted a study of the bitcoin exchange rate and its optimal FX futures hedge. Three hedge ratios calculated by different approaches were compared in order to identify the optimal one. Results suggested that, in terms of hedging effectiveness, the dynamic model was competitive with the naïve and conventional methods implemented in the static model. However, an arbitrage strategy that relies on the bitcoin exchange rate appears rather primitive due to the one-at-a-time nature of the unidirectional exchanges; for example, after trading Euros for U.S. dollars in the bitcoin market, speculators must wait for another arbitrage opportunity to change their dollars back to Euros. To do otherwise would mean facing the problem of a limited budget and the risk of holding dollars. This study proposes a triangular arbitrage in which investors who sell Euros and buy U.S. dollars in the bitcoin market execute a reverse transaction in the FX spot market and compares triangular arbitrage with different assets from the bitcoin and FX markets. In employing such a triangular arbitrage, speculators want to minimize their exposure to Euro exchange rate risk by going short in $b$ Euros-worth of futures contracts. This issue is addressed by optimizing a specific utility function that requires the second moment of two returns—the return of the triangular arbitrage and the return of futures contracts— to be measured. This study employs a time-dependent bivariate GACCH model to measure the joint density of the two returns. Moreover, based on the proposed model, three important time-varying series – the covariance matrix, optimal hedge ratio, and correlation – are forecast on a one-step-ahead basis.

The remainder of the paper is divided into two sections: Section 2 introduces the data set and explains the methodology; Section 3 gives empirical results and conclusions.

## 2 Data and methodology

The data set in this study consists of four daily closing prices series: (1) the USD bitcoin index , (2) the Euro bitcoin index, (3) the USD/EUR spot rate, and (4) the USD/EUR European-style FX monthly futures contract. The data period extends from 21 April 2014 to 21 September 2018. After merging the data by date, we were left with 1111 observations for each series. The last 110 observations were reserved for out-sample forecasting.

### 2.1 Calculations of the bitcoin exchange rate and the triangular arbitrage

Equation (1) is used to calculate the bitcoin USD/EUR exchange rate, $(USD/EUR)_{BX}$ :

$$(USD/EUR)_{BX} = \frac{USD/BTC}{EUR/BTC} \quad (1)$$

where $USD/BTC$ and $EUR/BTC$ are the prices of bitcoins in U.S. dollars and Euros, respectively (Nan & Kaizoji, 2018a).

The triangular arbitrage is expressed by

$$Triangular\ arbitrage = \frac{USD/BTC}{EUR/BTC} \times EUR/USD \qquad (2)$$

where EUR/USD=1/(USD/EUR) is the reciprocal of the USD/EUR spot rate.

The logarithmic return of the triangular arbitrage is $R_{TA}$ , given by

$$R_{TA} = \log\left(\frac{USD/BTC}{EUR/BTC} \times \frac{1}{USD/EUR}\right) = BX_t - SP_t \qquad (3)$$

where $BX_t$ and $SP_t$ denote the logarithm of the USD/EUR bitcoin exchange rate and the logarithm of the USD/EUR spot rate, respectively.

## 2.2 The futures hedge and the optimal hedge ratio

The reason for incorporating a futures contract into a portfolio is to reduce value fluctuations. The logarithmic return of the hedged portfolio composed of $l_{TA}$ units of a long triangular arbitrage position and $l_{Fu}$ units of a short futures position is given by

$$R_{HP} = l_{TA}\, R_{TA} - l_{Fu}\, R_{Fu} \qquad (4)$$

where $R_{HP}$, $R_{TA}$ and $R_{Fu}$ denote the logarithmic returns of the hedged portfolio, the triangular arbitrage, and the futures contract, respectively. The logarithmic return is normalized by making the units of the triangular arbitrage position equal to unity and has the form $R_{HP}/P_{TA} = R_{TA} - (l_{Fu}/l_{TA})R_{Fu}$. The coefficient $c = l_{Fu}/l_{TA}$ is called the hedge ratio.

The optimal hedge ratio is obtained by optimizing a specific objective function. One of the often-used objective functions is the variance of the portfolio, which, in this case, has the form

$$Var(R_{HP}) = l_{TA}^2\, Var(R_{TA}) + l_{Fu}^2\, Var(R_{Fu}) - 2\, l_{TA}\, l_{Fu}\, Cov(R_{TA}, R_{Fu}) \qquad (5)$$

By minimizing the variance of the portfolio (or the portfolio risk) in terms of the hedge ratio $c = P_{Fu}/P_{TA}$, the quadratic function has a minimum at which

$$c^* = \frac{l_{Fu}}{l_{TA}} = \frac{Cov(R_{TA}, R_{Fu})}{Var(R_{Fu})} = \rho\, \frac{\sigma_{TA}}{\sigma_{Fu}} \qquad (6)$$

where $c^*$ denotes the optimal hedge ratio using the minimum variance method, $\rho$ denotes the correlation coefficient between $R_{TA}$ and $R_{Fu}$, and $\sigma_{TA}$ and $\sigma_{Fu}$ denote the standard deviation of $R_{TA}$ and $R_{Fu}$, respectively.

As the minimum variance hedge ratio does not take into account the return of the hedged portfolio, the mean-variance hedge ratio is proposed as a remedy to this problem (Hsln et al., 1994). The mean-variance expected utility function is given by

$$EU(\boldsymbol{R}_t) = E(\boldsymbol{R}_t) - \gamma\, Var(\boldsymbol{R}_t) \quad (7)$$

where $\boldsymbol{R}_t$ is the $2 \times 1$ dimensional return vector containing two entries, $R_{TA}$ and $R_{Fu}$ ; $\gamma > 0$ represents the risk aversion parameter. Speculators maximize their expected utility by solving equation (7) for ; the optimal mean-variance hedge ratio $c^{**}$ is shown by

$$c^{**} = -\left[\frac{E(R_{Fu})}{2\,\gamma\, Var(R_{Fu})} - \frac{Cov(R_{TA}, R_{Fu})}{Var(R_{Fu})}\right]. \quad (8)$$

When $(R_{Fu}) = 0$ , indicating that the futures series follows a martingale, Equation (8) reduces to Equation (6).

### 2.3 Measuring the joint density and the time-dependent variance-covariance matrix

To calculate $c^*$ in Equation (6) and $c^{**}$ in Equation (8), it is necessary to model the joint density of $R_{TA}$ and $R_{Fu}$. Time-varying variances and covariances are more attractive to practitioners than static ones, as practitioners alter their portfolio over time rather than holding a fixed position throughout the period. Moreover, financial return series often show a clustering of volatility after a tranquil period. This heteroskedastic characteristic appears to cause the probability density of the return to show excess kurtosis and fat tails (Baillie & Myers, 1991). A wide range of multivariate GARCH models have been devised to estimate density with the aforementioned features. This study proposes using the bivariate Dynamic Conditional Correlation Generalized Autoregressive Conditional Heteroskedastic (DCC-GARCH) model introduced by Engle & Sheppard (2001) and Engle (2002). Using the DCC (1, 1) – GARCH (1, 1) model, our specification is given by

$$\boldsymbol{e}_t|\Omega_{t-1} \sim MN(0, \boldsymbol{H}_t) \tag{9}$$

$$\boldsymbol{H}_t \equiv \boldsymbol{D}_t \mathbf{P}_t \boldsymbol{D}_t \tag{10}$$

$$\boldsymbol{D}_t^2 = diag(\boldsymbol{\omega}) + diag(\boldsymbol{\alpha})\, \boldsymbol{e}_t\, \boldsymbol{e}_t' + diag(\boldsymbol{\beta})\boldsymbol{D}_{t-1}^2 \tag{11}$$

$$\boldsymbol{\varepsilon}_t = \boldsymbol{D}_t^{-1}\boldsymbol{e}_t \tag{12}$$

$$\boldsymbol{Q}_t = \overline{\boldsymbol{Q}}(\boldsymbol{u}' - \boldsymbol{\Phi} - \boldsymbol{\Psi}) + \boldsymbol{\Phi}\boldsymbol{\varepsilon}_{t-1}\, \boldsymbol{\varepsilon}_{t-1}' + \boldsymbol{\Psi}\, \boldsymbol{Q}_{t-1} \tag{13}$$

$$\mathbf{P}_t = diag\left(\boldsymbol{Q}_t^{1/2}\right)^{-1} \boldsymbol{Q}_t\, diag\left(\boldsymbol{Q}_t^{1/2}\right)^{-1} \tag{14}$$

In Equation (9), $\boldsymbol{e}_t$ is the $2 \times 1$ dimensional residual vector comprising the two residuals produced by a regression in which each of the returns, $R_{TA}$ and $R_{Fu}$ , has been inserted into the ARMA (1, 1) model. Equation (9) indicates that after the time-varying means are removed from the two returns, the residual vector is assumed to follow a multivariate normal distribution with mean zero and covariance matrix $\boldsymbol{H}_t$. The $\Omega_{t-1}$ term denotes the previous information set conditioned on $\boldsymbol{e}_t$. Equation (10) shows that $\boldsymbol{H}_t$ can be decomposed into $\boldsymbol{D}_t\mathbf{P}_t\boldsymbol{D}_t$, where $\boldsymbol{D}_t$ is the diagonal matrix with time-dependent standard deviations as described in (11) in the diagonal, and $\mathbf{P}_t$ is the time-dependent conditional correlation matrix. Equation (11) captures the univariate GARCH effect for each return. The standard deviation matrix $\boldsymbol{D}_t^{-1}$ is then used to standardize the residual vector $\boldsymbol{e}_t$ to obtain the standardized innovation vector $\boldsymbol{\varepsilon}_t$**,** as specified in (12). In equation (13), $\overline{\boldsymbol{Q}}$ denotes the unconditional correlation matrix of $\boldsymbol{\varepsilon}_t$ resulting from the first stage estimation, and $\boldsymbol{u}$ is a vector of unities. The specification in (13) indicates that the proxy process $\boldsymbol{Q}_t$ has exponential smoothing structure related not only to its one-period-ahead lag but also the lagged standardized deviation. The conditional correlation matrix $\mathbf{P}_t$ is then calculated through rescaling $\boldsymbol{Q}_t$ as shown in (14).

### 2.4 Forecasting

Normally, the r-step-ahead forecast of a multivariate GARCH (1, 1) has the form

$$\boldsymbol{D}_{t+r} = \sum_{i=1}^{r-2} diag(\boldsymbol{\omega})\,[diag(\boldsymbol{\alpha}) + diag(\boldsymbol{\beta})]^{i} + [diag(\boldsymbol{\alpha}) + diag(\boldsymbol{\beta})]^{r-1}\boldsymbol{D}_{t+1}, \tag{15}$$

However, due to the non-linearity of the DCC evolution process shown in (13) and (14), the multi-step forecast of correlation does not have a direct solution. In such a case, Engle & Sheppard (2001) suggest two ways to make approximations.

One technique said to provide for the least bias is that $\overline{\boldsymbol{Q}} \approx \overline{\boldsymbol{P}}$ and $E_t[\boldsymbol{Q}_{t+1}] = E_t[\boldsymbol{P}_{t+1}]$. Consider the r-step-ahead evolution of the proxy process

$$\boldsymbol{Q}_{t+r} = \overline{\boldsymbol{Q}}(\boldsymbol{u}' - \boldsymbol{\Phi} - \boldsymbol{\Psi}) + \boldsymbol{\Phi}\, E_t[\boldsymbol{\varepsilon}_{t+r-1}\, \boldsymbol{\varepsilon}'_{t+r-1}] + \boldsymbol{\Psi}\, \boldsymbol{Q}_{t+r-1} \tag{16}$$

and its approxiamted r-step-ahead forecast given by

$$E_t[\boldsymbol{P}_{t+r}] = \sum_{i=0}^{r-2} (\boldsymbol{u}' - \boldsymbol{\Phi} - \boldsymbol{\Psi})\, \overline{\boldsymbol{P}}(\boldsymbol{\Phi} + \boldsymbol{\Psi})^{i} + (\boldsymbol{\Phi} + \boldsymbol{\Psi})^{r-1}\boldsymbol{P}_{t+1} \tag{17}$$

where $\overline{\boldsymbol{P}}$ denotes the constant correlation matrix of returns. In practice, $\overline{\boldsymbol{P}}$ is often used as the covariance matrix in place of the correlation matrix since the standardized innovation is usually unobservable (Engle & Sheppard, 2001).

In this study, to minimize bias, a rolling one-step-ahead approach is used, since, for r-step-ahead forecasting, forecast error based on a fixed value for $\overline{\boldsymbol{P}}$ becomes quite large when when the value of r is large.

## 3 Empirical results and conclusion

### 3.1 Statistical characteristics of returns

The USD/EUR bitcoin exchange rate is computed by (1). Figure 2 plots the respective time series: the bitcoin exchange rate series, $BX_t$; the FX spot rate series, $SP_t$; and the FX futures rate series, $FU_t$, all transformed into natural logarithms. As Figure 2 indicates, $BX_t$ shows an intertwining with $SP_t$ but seemingly has more spikes. Surprisingly, $BX_t$ appears not to deviate far from $SP_t$ , even during the period at the end of 2018 when the bitcoin prices of USD and EUR skyrocketed. As for $FU_t$, it runs above the $BX_t$ and $SP_t$ series; however, the discrepancy tends to narrow, indicating a diminishing risk premium.

Figure 3 plots the six logarithmic return series: (1) the return of the USD/BTC index, $R_{USD/BTC}$; (2) the return of the EUR/BTC index, $R_{EUR/BTC}$; (3) the return of the USD/EUR bitcoin exchange rate, $R_{BX}$; (4) the return of the USD/EUR spot rate, $R_{SP}$; (5) the return of USD/EUR futures rate, $R_{FU}$, and (6) the return of the triangular arbitrage, $R_{TA}$. Although all of these return series appear to oscillate around their mean and behave rather noisily, $R_{TA}$ presents a cyclic fluctuating pattern that indicates short-run arbitrage opportunities.

Figure 4 plots the sample probability densities of the six logarithmic return series. As Panel (a) and Panel (b) of Figure 3 show, the graphs of the densities of $R_{USD/BTC}$ and $R_{EUR/BTC}$ have long and fat left tails extending negatively beyond -0.2 on the x-axis. These phenomena suggest the potential for more than 20% daily losses in terms of either the U.S. dollar or the Euro when investors take a positioning trading strategy on bitcoins. That both left tails are seemingly longer than the right tails appears to support Dong & Dong's (2015) finding that bitcoin investment presents higher risk and lower return. In contrast, Panel (c) of Figure 3 suggests a reduction in daily losses to around 6% or so when using the bitcoin to make currency exchanges. The triangular arbitrage strategy appears to improve the currency exchange strategy by means of a more centered and symmetric probability density.

The summary statistics in Table 1 affirm our conjectures based on the observed plots. First, bitcoin trading had a much wider range of returns than did currency exchange in either the FX market or the bitcoin market. This result indicates that bitcoin investors employing a buy-and-hold strategy face exceedingly significant losses or profits over time. Second, bitcoin-based currency exchange strategies had much less static variance during the sample period. The reduction in the unconditional variances is substantial when speculators perform bitcoin-based currency exchange strategies, while the triangular arbitrage reduces the variance as compared to the bitcoin exchange rate strategy. Third, the density of the return on bitcoin investment shows negative skewness, suggesting that most bitcoin holders can expect only a slight profit, at the price of a very large loss. Furthermore, all six series show excess kurtosis; hence, they all lead to rejecting the assumption of normality (the Jarque-Bera test) in a very significant way. Usually, non-normal densities of returns are the result of weak dependence in the return series. Here, the Student's *t* distribution functions better than the normal distribution hypothesis for the GARCH family (Baillie & Myers, 1991). An interesting phenomenon is that the density of $R_{TA}$ shows a very high excess kurtosis, which may imply a strong dependence in the return series. Moreover, since the data are more centrally distributed, the areas in the tails of the distribution decrease, indicating a reduction in risk.

### 3.2 The DCC-GARCH model and the estimated optimal hedge ratio

Time-series features of the six series of returns are summarized in Table 2. First, to test stationarity, the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) and Elliott-Rothenberg-Stock (ERS) tests were conducted. The null hypothesis for the KPSS test of stationarity could not be rejected in all cases using models with a lag of 21. Meanwhile, the null of a unit root process for the ERS test was rejected in all cases at the 1% significance level. Based on these results, all six returns appear to be stationary—i.e., a long-run martingale process. The Ljung-box test (or the Q-statistic) using a lag of 20 indicated that there was no serial correlation in $R_{USD/BTC}$, $R_{EUR/BTC}$, $R_{SP}$, and $R_{FU}$; however, it identified serial correlations in $R_{BX}$ and $R_{TA}$. The order of the ARMA model is suggested by the AIC criterion. For $R_{TA}$, although AIC recommended the ARMA (2, 1) model, the coefficient of the second order autoregressive term is not significant. Thus, the ARMA (1, 1) model may be appropriate for parsimony.

Based on the obtained information, we propose a framework to model the joint density of the return of triangular arbitrage, $R_{TA}$, and the return of futures, $R_{FU}$, i.e., to capture the time-varying conditional variance matrix and the time-varying conditional correlation. The proposed framework combines the ARMA (1, 1) model with the DCC (1, 1)-bivariate GARCH (1, 1) model. For the univariate GARCH model employed in the first step, a Student's t distribution is assumed, whereas the multivariate normal distribution is assumed in the second step. A total of 1000 observations were used to fit the proposed model; results are presented in Table 3.

First, the ARMA (1, 1) model was used to model autoregression in the return series, especially in $R_{TA}$. All three coefficients related to $R_{TA}$ were significant. Comparing the values of these coefficients with those estimated from the univariate ARMA (1, 1) model that was applied to $R_{TA}$, the values are quite similar. The residuals from the univariate ARMA (1, 1) model applied to $R_{TA}$ passed the Ljung-Box test using a lag of 20, which shows ARMA (1, 1) is sufficient for capturing the autoregressive effect. Second, the GARCH (1, 1) model with Student's $t$ distribution as the underlying assumption was used to measure the volatility clustering effect. Except for the constant terms, all coefficients were significantly different from zero. The 0.9957 sum of $\alpha_{TA}$ and $\beta_{TA}$ and the 0.9970 sum of $\alpha_{FU}$ and $\beta_{FU}$ suggest a near integrating process for the volatility of the triangular arbitrage returns and the FX futures returns, respectively. The shape parameters for the Student's *t* distribution were all significant: $v_{TA} = 3.2055$ implies much heavier tails in the density of $R_{TA}$ than in the case of $R_{FU}$ , where $v_{FU} = 6.4826$. Finally, the 0.9324 sum of $\varphi + \psi$ tends to suggest a comparatively persistent conditional correlation process.

Figure 5 plots the conditional variance of the return of the triangular arbitrage, $h_{tt,t}$, the conditional variance of the return of the FX futures, $h_{ff,t}$, and the conditional covariance of the two returns, $h_{tf,t}$. The time-varying conditional optimal hedge ratio is calculated as $c_t^* = h_{tf,t}/h_{ff,t}$ according to Equation (6).

Figure 6 shows a plot of the conditional optimal hedge ratio series, $c_t^*$, and the conditional correlation series, $P_t$ . In Panel (a) of Figure 6, the dashed line is the unconditional optimal hedge ratio, $c_{OLS}^* = -0.2510$, obtained from the OLS regression. By comparison, the conditional optimal hedge ratio has mean -0.32 and varies from -2.5065 to -0.1221. The dashed line in Panel (b) of Figure 6 is the unconditional correlation between $R_{TA}$ and $R_{FU}$. The unconditional correlation value is -0.1760, whereas the conditional correlation series varies from -0.3435 to -0.1694.

### 3.3 Rolling one-step-ahead forecasting

The estimated coefficients of our specification—the ARMA (1, 1) plus the DCC-bivariate GARCH—were used to produce rolling one-step-ahead forecasts. The rolling window was fixed at a length of 1000. This approach makes a one-step-ahead forecast, and then adds a new observation to the end of the window and removes the first observation. In this fashion, the process was repeated 110 times until all 110 bits of data were exhausted.

The forecasted conditional variances and covariance, $h'_{tt,t}$, $h'_{ff,t}$, and $h'_{tf,t}$, and the forecasted conditional correlation $P'_t$ are plotted in Figure 7. Figure 8 combines the conditional optimal hedge ratio $c_t^*$ obtained from our sample (black line) and the forecasted optimal hedge ratio (red dashed line) into a single plot.

### 3.4 Comparison

To make a comparison, the unconditional variance (multiplied by the number of the observations) and Value-at-Risk (VaR) were used as the criteria to illustrate the performance of all the arbitrage strategies. Table 4 gives a summary of results for the sample of 1000 observations. As shown, the returns of the triangular arbitrage and the returns of the hedged portfolio have much smaller unconditional variances than the returns of USD/BTC and EUR/BTC, although they are not the smallest when compared to the FX spot and futures returns. The returns of the USD/EUR bitcoin exchange rate also show a smaller variance than the returns of USD/BTC and EUR/BTC. The VaR criteria indicate that the hedge portfolio appears to have the least negative value of the returns at the high quantile 99.5 using either the Historical approach or the Cornish-Fisher (denoting Modified-) approach, which is modified to adapt to the leptokurtic density of returns. For $R_{HP}$, the one-day losses at the 0.5% confidence level are -1.95% and -4.11%, respectively, according to the historical and the modified approaches. In contrast, $R_{USD/BTC}$ has daily losses of -14.23% and -19.62, respectively; for $R_{EUR/BTC}$ , the losses are -14.35% and 20.07%, respectively.

Table 5 gives the results using the same criteria as in Table 4 but involves a forecast period of 18 April 2017 to 21 September 2018. The returns of the hedged portfolio using the forecasted optimal hedge ratio $R_{HP}$ shows the lowest daily risks with respect to the VaR approaches. An interesting outcome is that $R_{TA}$ and $R_{HP}$ during the observation period have smaller variances than the returns from the FX market.

### 3.5 Conclusion

Based on empirical results, this paper finds that bitcoin-based currency exchange strategies are competitive with bitcoin trading in terms of risk management. In particular, the triangular arbitrage strategy using bitcoin as the medium of exchange appears to be most attractive due to the existence of arbitrage opportunities and FX futures hedging. The ARMA plus DDC-GARCH model is suggested to measure joint density, to capture the time-varying conditional covariance matrix and correlation, and to produce one-step-ahead rolling window forecasts. Hence, a dynamic future hedged portfolio was formed, the return of which was then compared to the returns of the bitcoin and FX rates. The empirical and forecasted results tend to show that the hedged portfolio approach is superior to the others.

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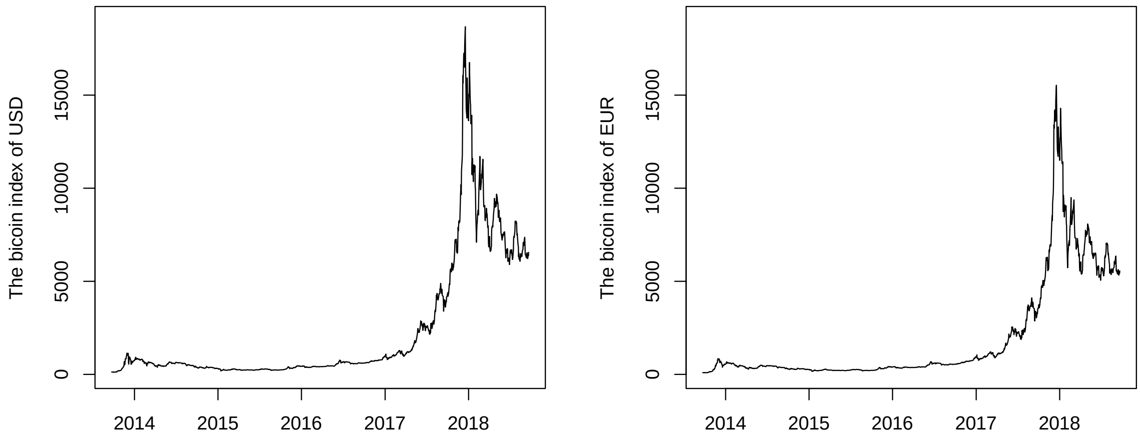


**Figure 1.** Bitcoin price indices of USD and EUR

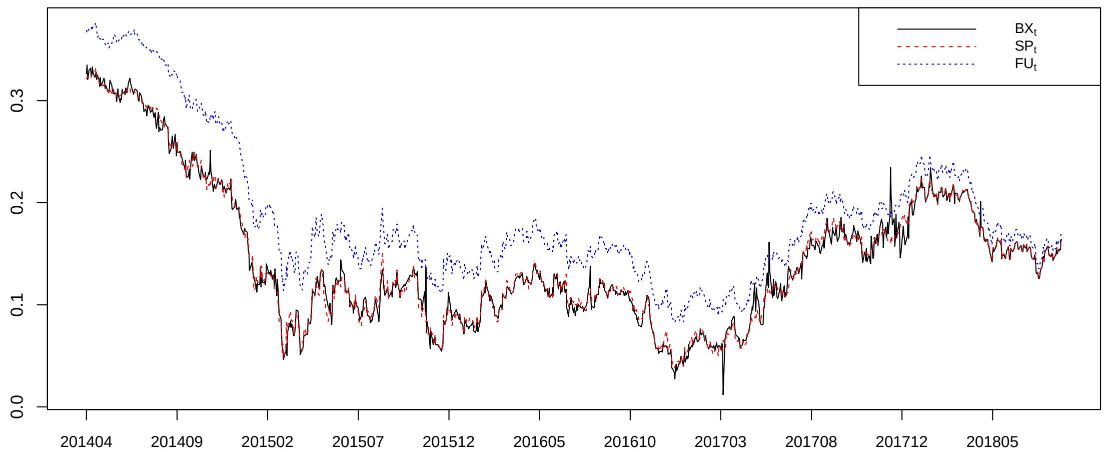


**Figure 2.** The bitcoin exchange rate of USD/EUR ($BX_t$), the FX spot of USD/EUR ($SP_t$) and the FX futures of USD/EUR ($FU_t$). All series are in natural logarithm form.

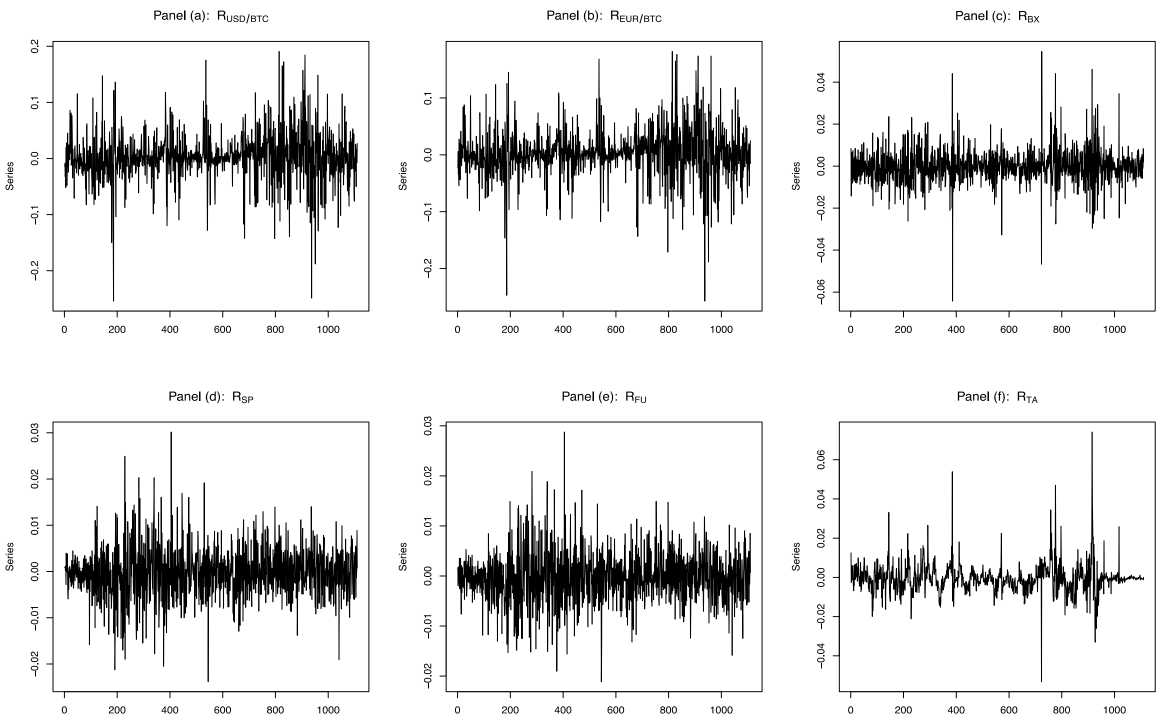


**Figure 3.** The return series of the six assets (or portfolios): the returns of USD/BTC ($R_{USD/BTC}$), the returns of EUR/BTC ($R_{EUR/BTC}$), the returns of the USD/EUR bitcoin exchange rate ($R_{BX}$), the returns of the FX spot rates ($R_{SP}$), the returns of the FX futures rate ($R_{FU}$), and the returns of the triangular arbitrage ($R_{TA}$).

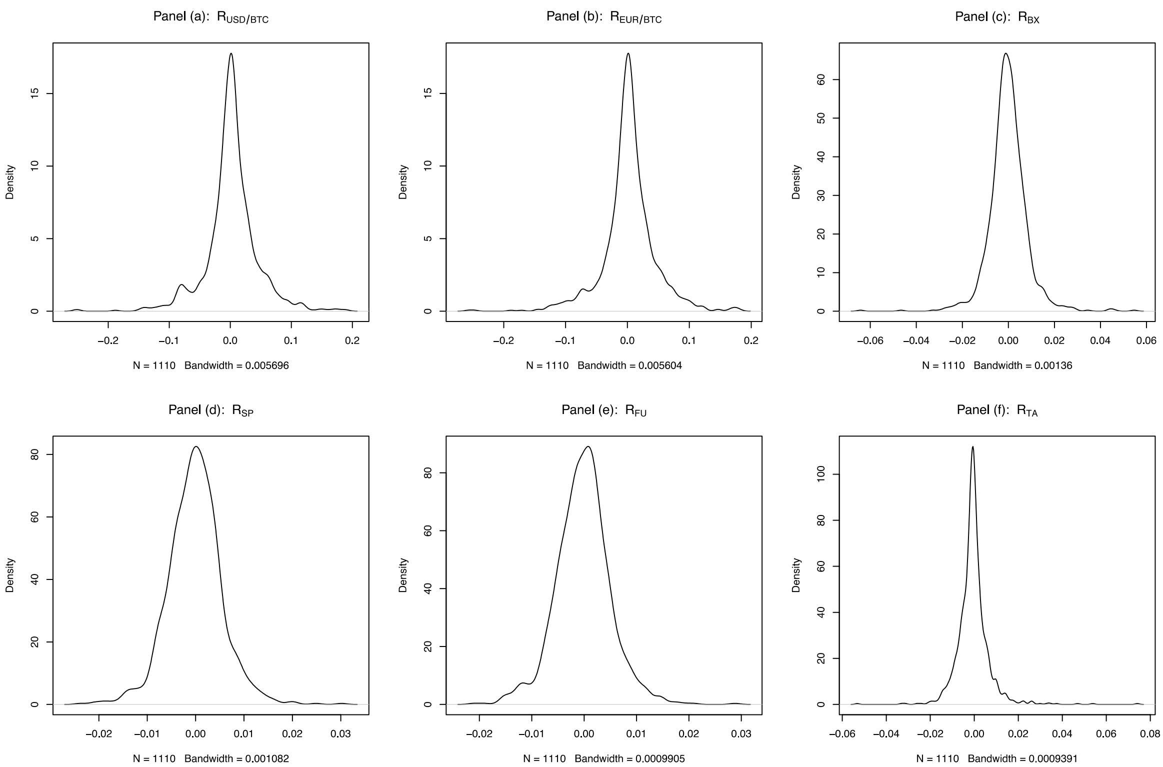


**Figure 4.** The sample probability densities of the six return-series.

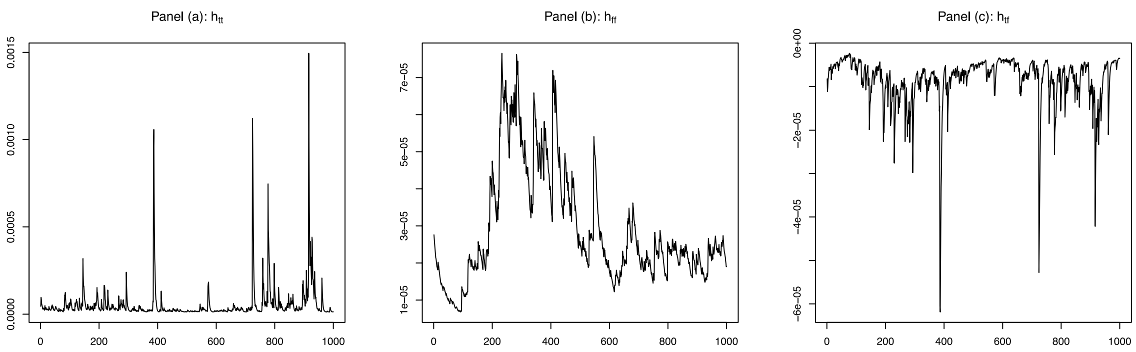


**Figure 5.** The conditional variance of the returns of the triangular arbitrage ($h_{tt,t}$), the conditional variance of the returns of the futures rate ($h_{ff,t}$), and the conditional covariance ($h_{tf,t}$).

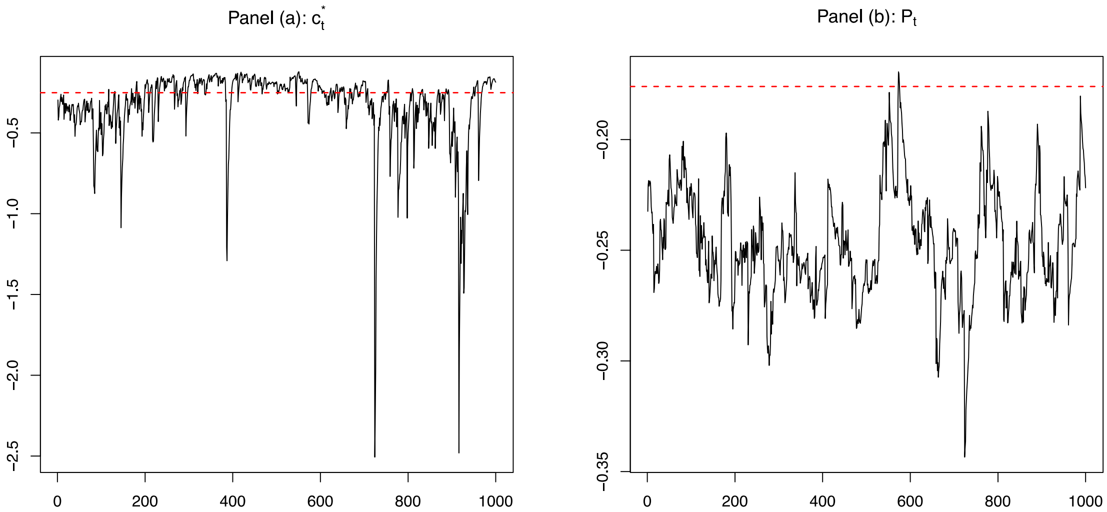


**Figure 6.** The conditional optimal hedge ratio series ($c_t^*$) and the conditional correlation series ($P_t$). The red dashed line in Panel (a) is the optimal hedge ratio obtained from the OLS method ($c_{OLS}$); the red dashed line in Panel (b) is the unconditional correlation coefficient.

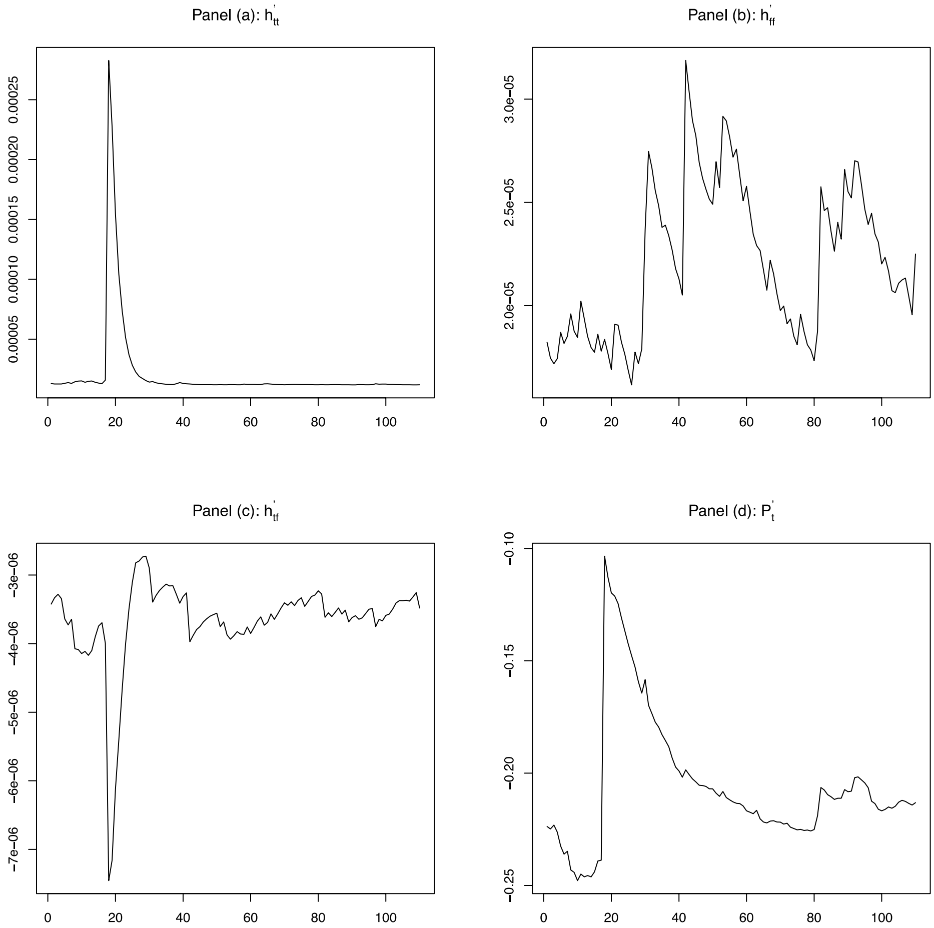


**Figure 7.** The forecasted conditional variances and covariance and the forecasted conditional correlation based on the ARMA plus DCC- GARCH model.

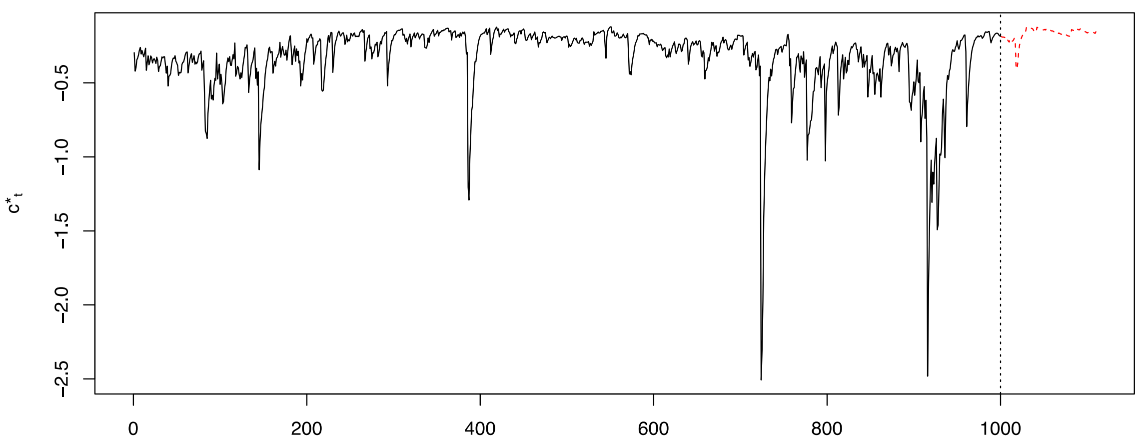


**Figure 8.** The sample conditional optimal hedge ratio (black line) and the forecasted conditional optimal hedge ratio (red dashed line).

**Table 1.** Summary statistics

| Returns | Mean | Minimum | Maximum | S.D. | Skewness | Kurtosis | Normality |
|---|---|---|---|---|---|---|---|
| $R_{USD/BTC}$ | 0.0023 | -0.2542 | 0.1909 | 0.0430 | -0.23 | 4.44 | 928.05** |
| $R_{EUR/BTC}$ | 0.0025 | -0.2573 | 0.1818 | 0.0427 | -0.33 | 4.63 | 1015.00** |
| $R_{BX}$ | -0.0001 | -0.0643 | 0.0546 | 0.0084 | 0.16 | 8.21 | 3136.90** |
| $R_{SP}$ | -0.0001 | -0.0238 | 0.0301 | 0.0056 | 0.12 | 2.15 | 219.26** |
| $R_{FU}$ | -0.0001 | -0.0212 | 0.0288 | 0.0052 | 0.15 | 1.87 | 167.67** |
| $R_{TA}$ | -0.0005 | -0.0532 | 0.0741 | 0.0074 | 1.59 | 17.66 | 14949** |

**Note:** The sample period contains 1000 observations. Kurtosis refers to the excess kurtosis. Normality refers to the Jarque-Beta test.

**Table 2.** Time-series features

| Returns | KPSS | ERS | Ljung-Box | ARMA |
|---|---|---|---|---|
| $R_{USD/BTC}$ | 0.14 | -8.16** | 14.27 | (1, 1) |
| $R_{EUR/BTC}$ | 0.12 | -7.15** | 12.20 | (1, 1) |
| $R_{BX}$ | 0.08 | -4.17** | 93.91** | (2, 3) |
| $R_{SP}$ | 0.09 | -9.20** | 18.25 | (0, 0) |
| $R_{FU}$ | 0.09 | -8.03** | 15.04 | (0, 0) |
| $R_{TA}$ | 0.04 | -4.85** | 559.37** | (2, 1) |

**Note: Ljung-Box** refers to the Q-statistic using a lag of 20.

**Table 3.** Estimation of the bivariate ARMA plus DCC-GARCH model

| ARMA (1, 1) | | Bivariate GARCH (1, 1) | | DCC (1, 1) and others | |
|---|---|---|---|---|---|
| $\mu_{TA}$ | -0.0009** | $\omega_{TA}$ | 0.0000 | $\varphi$ | 0.0111 |
| | (0.0003) | | (0.0000) | | (0.0165) |
| $\mu_{FU}$ | -0.0002 | $\omega_{FU}$ | 0.0000 | $\psi$ | 0.9231** |
| | (0.0001) | | (0.0000) | | (0.0357) |
| $AR1_{TA}$ | 0.7905** | $\alpha_{TA}$ | 0.3553** | | |
| | (0.0402) | | (0.1156) | | |
| $AR1_{FU}$ | 0.4146 | $\alpha_{FU}$ | 0.0501** | $LL$ | 7650.927 |
| | (0.4111) | | (0.0073) | | |
| $MA1_{TA}$ | -0.3826** | $\beta_{TA}$ | 0.6404** | $AIC$ | -15.268 |
| | (0.0683) | | (0.1031) | | |
| $MA1_{FU}$ | -0.4520 | $\beta_{FU}$ | 0.9469** | | |
| | (0.4022) | | (0.0076) | | |
| | | $v_{TA}$ | 3.1055** | | |
| | | | (0.3837) | | |
| | | $v_{FU}$ | 6.4826** | | |
| | | | (1.2413) | | |

**Note:** $LL$ denotes Log-Likelihood, and $AIC$ denotes the Akaike Information Criterion for the framework.
** significant at 1%.

**Table 4** Comparison of assets or portfolios based on the unconditional variances and the VaR values in the sample period.

| Assets | Variances | Historical VaR (0.5%) | Modified VaR (0.5%) |
|---|---|---|---|
| $R_{USD/BTC}$ | 1.8478 | -0.1423 | -0.1962 |
| $R_{EUR/BTC}$ | 1.8203 | -0.1435 | -0.2007 |
| $R_{BX}$ | 0.0708 | -0.0272 | -0.0477 |
| $R_{SP}$ | 0.0310 | -0.0170 | -0.0185 |
| $R_{FU}$ | 0.0269 | -0.0151 | -0.0166 |
| $R_{TA}$ | 0.0548 | -0.0211 | -0.0476 |
| $R_{HP}$ | 0.0593 | -0.0195 | -0.0411 |

**Note:** VaR denotes the Value-at-Risk where Modified VaR concerns the Cornish-Fisher estimate of VaR. The sample period starts on 22 April 2014 and ends on 17 April 2017; it includes 1000 observations.

**Table 5** Comparison of assets or portfolios based on the unconditional variances and the VaR values in the forecasted period.

| Assets | Variances | Historical VaR (5%) | Modified VaR (5%) |
|---|---|---|---|
| $R_{USD/BTC}$ | 0.1557 | -0.0658 | -0.0639 |
| $R_{EUR/BTC}$ | 0.1581 | -0.0599 | -0.0629 |
| $R_{BX}$ | 0.0040 | -0.0079 | -0.0072 |
| $R_{SP}$ | 0.0024 | -0.0073 | -0.0086 |
| $R_{FU}$ | 0.0023 | -0.0071 | -0.0084 |
| $R_{TA}$ | 0.0008 | -0.0019 | |
| $R_{HP}$ | 0.0009 | -0.0027 | -0.0065 |

**Note:** VaR denotes the Value-at-Risk where Modified VaR concerns the Cornish-Fisher estimate of VaR. The forecasted period starts on 18 April 2017 and ends on 21 September 2018; it includes 110 observations. The Modified VaR is blank in the $R_{TA}$ row because of the unreliable result (inverse risk) produced by the calculation.