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Research article

Survival analysis of an impulsive stochastic delay logistic model with Lévy jumps

Chun Lu[1], **Bing Li**[2,*], **Limei Zhou**[1] **and Liwei Zhang**[3]

[1] Department of Mathematics, Qingdao University of Technology, Qingdao, 266520, China
[2] School of Mathematical Science, Harbin Normal University, Harbin, 150025, China
[3] School of Civil Engineering, Qingdao University of Technology, Qingdao, 266520, China

* **Correspondence:** Email: leeicer@126.com.

Abstract: This paper studies a stochastic delay logistic model with Lévy jumps and impulsive perturbations. We show that the model has a unique global positive solution. Sufficient conditions for extinction, non-persistence in the mean, weak persistence, stochastic permanence and global asymptotic stability are established. The threshold between weak persistence and extinction is obtained. The results demonstrate that impulsive perturbations which may represent human factor play an important role in protecting the population even if it suffers sudden environmental shocks that can be discribed by Lévy jumps.

Keywords: persistence; Lévy jump; impulsive perturbation; logistic model; infinite delay

1. Introduction

In the real world, "all species should exhibit time delay" [1]. As is known to all, logistic model with delay is one of the most important and classical model in mathematical biology. The deterministic logistic model with infinite delay is described by the differential equation:

$$\frac{dy(t)}{dt} = y(t)\Big[b - a_1 y(t) + a_2 y(t-\tau) + a_3 \int_{-\infty}^{0} y(t+\theta)d\zeta(\theta)\Big], \tag{1.1}$$

where $y(t)$ is the population size at time t, b denotes the intrinsic growth rate, τ represents the time delay and $\zeta(\theta)$ is a probability measure on $(-\infty, 0]$. Model (1.1) and its various forms have been investigated extensively (see [1]– [6]). Particularly, Gopalsamy [2] and Kuang [6] have obtained the classical result: if $b > 0$ and $a_1 > a_2 + a_3$, then the positive equilibrium of model (1.1) is globally asymptotically stable.

Biological populations are inevitably subject to environmental noises which are the important component in an ecosystem(see [7]– [16]). And time delay and system uncertainty are commonly encountered and are often the sources of instability [4]. Considering the effect of stochastic factors for infinite delay systems have attracted great attentions in recent years(see [9]– [12], [14]– [16]). Wu et al. [14] studied the effects of environmental noise on the asymptotic properties of population model with infinite delay for the first time.

For the intrinsic growth rate b of model (1.1), there are many methods of introducing random perturbations into the models from biological and mathematical perspectives [17]– [25]. The corresponding non-autonomous system of (1.1) can be described as follows:

$$\begin{aligned} dy(t) =& y(t)\Big[b(t) - a_1(t)y(t) + a_2(t)y(t-\tau) + a_3(t)\int_{-\infty}^{0} y(t+\theta)d\zeta(\theta)\Big]dt \\ &+ \sigma_1(t)y^{1+\varrho}(t)dB(t) + y(t^-)\int_{\mathbb{Y}} \beta(u)\tilde{N}(dt,du), \end{aligned} \tag{1.2}$$

where $B(t)$ is standard Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$, $\sigma_1(t)$ is positive continuous bounded function on $\overline{R}_+ = [0, +\infty)$, ϱ is a constant, $y(t^-) = \lim_{s\uparrow t} y(s)$, $N(dt, du)$ is a real-valued Poisson counting measure with characteristic measure λ on a measurable subset $\mathbb{Y}$ of $\overline{R}_+$ with $\lambda(\mathbb{Y}) < +\infty$, $\tilde{N}(dt, du) = N(dt, du) - \lambda(du)dt$, $\beta(u)$ is bounded function and $\beta(u) > -1, u \in \mathbb{Y}$. Also, $B(t)$ is independent of $N(dt, du)$. Recently, Liu et al. [22] get the results of the extinction, non-persistence in the mean and weak persistence for the following model:

$$\begin{aligned} dy(t) =& y(t)\Big[b(t) - a_1(t)y(t) + a_2(t)y(t-\tau(t)) + a_3(t)\int_{-\infty}^{0} y(t+\theta)d\zeta(\theta)\Big]dt \\ &+ \sigma_1(t)y(t)d\omega_1(t) + \sigma_2(t)y^{1+\alpha}(t)d\omega_2(t) + \sigma_3(t)y^{\beta}(t-\tau(t))d\omega_3(t) \\ &+ \sigma_4(t)y(t)\int_{-\infty}^{0} y(t+\theta)d\zeta(\theta)d\omega_4(t) + y(t^-)\int_{\mathbb{Y}} \gamma(t,y)\tilde{N}(dt,dy), \end{aligned} \tag{1.3}$$

where α, β are positive constants, $\omega_i(t)$ are standard Brownian motions, $i = 1, 2, 3, 4$, and $N(dt, du)$ is a real-valued Poisson counting measure. Obviously, model (1.2) is a special case of model (1.3).

Moreover, impulsive stochastic differential equations have been considered extensively because impulsive perturbations can describe the activities of human exploitation, such as planting and harvesting (see [26]– [34]). Liu et.al [26] investigated the impact of impulsive perturbation in the stochastic model driven by Brownian motion for the first time. However, there are little attention on the effects of impulsive perturbation in the stochastic delay model driven by Lévy jumps. This is because the impulsive perturbations can not classified as Lévy jumps(see [35,36]). Motivated by the above, we will study the following stochastic delay logistic model with Lévy jumps and impulsive perturbations:

$$\begin{cases} dy(t) = y(t)\Big[b(t) - a_1(t)y(t) + a_2(t)y(t-\tau) + a_3(t)\int_{-\infty}^{0} y(t+\theta)d\zeta(\theta)\Big]dt \\ \qquad + \sigma_1(t)y^{1+\varrho}(t)dB(t) + y(t^-)\int_{\mathbb{Y}}\beta(u)\tilde{N}(dt,du), \quad t \neq t_k, \ K \in N, \\ y(t_k^+) - y(t_k) = J_k y(t_k), \quad k \in N, \end{cases} \tag{1.4}$$

where $y(t_k) = \lim_{t\to t_k^-} y(t), y(t_k^+) = \lim_{t\to t_k^+} y(t)$(see [37]), N denotes the set of positive integers, $0 < t_1 < t_2 \cdots$, $\lim_{k\to+\infty} t_k = +\infty$. The other parameters are as defined above.

The main aims of the work are to discuss how impulsive perturbations impact on the persistence and extinction of (1.4). We need to obtain the sufficient conditions of stochastic permanence and global asymptotic stability for model (1.4), which is not studied in [22]. For model (1.4) without Lévy jumps and impulsive perturbations, in Ref. [16], Lu et.al have investigated the stochastic permanence when the parameter $a_3(t) > 0$. In this paper we will discuss the global asymptotic stability of (1.4) without Lévy jumps and impulsive perturbations when the parameter $a_3(t) < 0$, which are good supplements for the deficiency of [16].

To proceed, we introduce the notations and definitions into here.

$$g^u = \sup_{t\in\bar{R}_+} g(t), \qquad g^l = \inf_{t\in\bar{R}_+} g(t), \qquad \langle g(t)\rangle = \frac{1}{t}\int_0^t g(s)ds,$$

$$g_* = \liminf_{t\to+\infty} g(t), \qquad g^* = \limsup_{t\to+\infty} g(t), \qquad R_+ = (0,+\infty).$$

The following definitions are commonly used and we list them here.

Definition 1. *The population $y(t)$ is said to be extinction if* $\lim_{t\to+\infty} y(t) = 0$.

Definition 2. *The population $y(t)$ is said to be non-persistence in the mean(see e.g., [38]) if* $\limsup_{t\to+\infty}\langle y(t)\rangle = 0$.

Definition 3. *The population $y(t)$ is said to be weak persistence(see e.g., Hallam and Ma [39]) if* $\limsup_{t\to+\infty} y(t) > 0$.

Definition 4. *The population $y(t)$ is said to be stochastic permanence [13] if for an arbitrary $\varepsilon > 0$, there are constants $\alpha_1 > 0, \alpha_2 > 0$ such that $\liminf_{t\to+\infty}\mathcal{P}\{y(t) \le \alpha_1\} \ge 1-\varepsilon$ and $\liminf_{t\to+\infty}\mathcal{P}\{y(t) \ge \alpha_2\} \ge 1-\varepsilon$.*

Definition 5. *Let $y(t)$ and $y^*(t)$ be two arbitrary solutions with initial values ξ and ξ^*, respectively. If $\lim_{t\to+\infty}|y(t) - y^*(t)| = 0$ a.s., then we say that solution of model* (1.4) *without Lévy jumps and impulsive perturbations is globally asymptotically stable.*

The rest of the paper is arranged as follows. In Section 2, we obtain the sufficient conditions for extinction, non-persistence in the mean, weak persistence and stochastic permanence, respectively. In section 3, we get the sufficient condition of global asymptotic stability of solution for model (1.4) without Lévy jumps and impulsive perturbation. The last section, we illustrate the main results with figure testification.

2. Persistence and extinction

In this section, we shall present the extinction and permanence of model (1.4). For simply research, we make the following assumptions:

(A1): Based on biological meanings, we assume that $1 + J_k > 0$.

(A2):$b(t)$, $a_1(t)$, $a_2(t)$, $a_3(t)$ and $\sigma_1(t)$ are continuous and bounded functions on $\overline{R}_+$, where $\overline{R}_+ = [0, +\infty)$. $a^l > 0$, $\sigma^l > 0$ and $\varrho > 0.75$.

(A3):There exists a positive constant c such that $|\ln(1 + \beta(u))| \leq c$ for $\beta(u) > -1$.

(A4): $\mu_{\mathbf{r}} = \int_{-\infty}^{0} e^{-2\mathbf{r}\theta} d\zeta(\theta) < +\infty$.

In this article, K denotes a positive constant and its value may be different in various conditions. Let the initial value ξ be positive and $\xi \in C_g$(see [2,22,40,41]) which is defined by

$$C_g = \{\varphi \in C((-\infty, 0]; R) : \| \varphi \|_{c_g} = \sup_{-\infty < s \leq 0} e^{\mathbf{r}s} |\varphi(s)| < +\infty\},$$

where $g(s) = e^{-\mathbf{r}s}, \mathbf{r} > 0$.

Lemma 1. *Suppose (A1)-(A4) hold. For model* (1.4)*, with any given initial value $\xi \in C_g$, there is a unique solution $y(t)$ on $t \in R$ with probability 1.*

Proof. Now consider the stochastic functional differential equation with infinite delay and Lévy jumps:

$$\begin{aligned} dz(t) =& z(t)\Big[b(t) - \prod_{0<t_k<t}(1 + J_k)a_1(t)z(t) + \prod_{0<t_k<t-\tau}(1 + J_k)a_2(t)z(t - \tau) \\ &+ a_3(t)\int_{-\infty}^{0} \prod_{0<t_k<t+\theta}(1 + J_k)z(t + \theta)d\zeta(\theta)\Big]dt \\ &+ \sigma_1(t)\Big(\prod_{0<t_k<t}(1 + J_k)\Big)^{\varrho} z^{1+\varrho}(t)dB(t) + z(t^-)\int_{\mathbb{Y}} \beta(u)\tilde{N}(dt, du) \end{aligned} \tag{2.1}$$

with the same initial value as model (1.4). Obviously, $\forall\ t \leq t_1$, $\prod_{0<t_k<t}(1 + J_k) = 1$ holds and $y(t) = \prod_{0<t_k<t}(1 + J_k)z(t), \forall\ t > 0$. By the work of Mao et al. [10], (2.1) has a unique maximal local solution, on $(-\infty, \tau_e)$, where τ_e is the explosion time. The rest of proof is standard and hence is omitted(see [26,32]). This completes the proof. □

Theorem 1. *Suppose (A1)-(A4) hold, if $G^* < 0$ and $\inf_{t\in\overline{R}_+}\{a_1(t) - a_2(t + \tau) - a_3^u\} \geq 0$, where $G^* = \limsup_{t\to+\infty} t^{-1}\Big[\sum_{0<t_k<t}\ln(1 + J_k) + \int_0^t b(s)ds\Big] - \int_{\mathbb{Y}}[\beta(u) - \ln(1 + \beta(u))]\lambda(du)$, then population $y(t)$ modeled by* (1.4) *will go to extinction almost surely (a.s.).*

Proof. For (2.1), using the Itô's formula, we get

$$d\ln z = \frac{dz}{z} - \frac{(dz)^2}{2z^2}$$

$$=\Big[b(t)-\prod_{0<t_k<t}(1+J_k)a_1(t)z+\prod_{0<t_k<t-\tau}(1+J_k)a_2(t)z(t-\tau)$$
$$+a_3(t)\int_{-\infty}^{0}\prod_{0<t_k<t+\theta}(1+J_k)z(t+\theta)d\zeta(\theta)-\frac{\Big(\prod_{0<t_k<t}(1+J_k)\Big)^{2\varrho}\sigma^2(t)z^{2\varrho}(t)}{2}$$
$$-\int_{\mathbb{Y}}(\beta(u)-\ln(1+\beta(u))\lambda(du)\Big]dt+\sigma_1(t)\Big(\prod_{0<t_k<t}(1+J_k)\Big)^{\varrho}z^{\varrho}(t)dB(t)$$
$$+\int_{\mathbb{Y}}\ln(1+\beta(u))\tilde{N}(dt,du).$$

Integrating both sides from 0 to t, where $t\in[0,t_1]$ or $t\in(t_k,t_{k+1}],k=1,2,\cdots$, we get

$$\ln z(t)-\ln z(0)=\int_0^t\Big[b(s)-\prod_{0<t_k<s}(1+J_k)a_1(s)z(s)$$
$$+\prod_{0<t_k<s-\tau}(1+J_k)a_2(s)z(s-\tau)+a_3(s)\int_{-\infty}^{0}\prod_{0<t_k<s+\theta}(1+J_k)z(s+\theta)d\zeta(\theta)$$
$$-\frac{(\prod_{0<t_k<s}(1+J_k))^2\sigma^2(s)z^{2\varrho}(s)}{2}\Big]ds$$
$$+\int_0^t\Big(\prod_{0<t_k<s}(1+J_k)\Big)^{\varrho}\sigma_1(s)z^{\varrho}(s)dB(s)-t\int_{\mathbb{Y}}[\beta(u)-\ln(1+\beta(u))]\lambda(du)$$
$$+\int_0^t\int_{\mathbb{Y}}\ln(1+\beta(u))\tilde{N}(ds,du).$$

Then

$$\ln z(t)-\ln z(0)=\int_0^t\Big[b(s)-a_1(s)y(s)+a_2(s)y(s-\tau)$$
$$+a_3(s)\int_{-\infty}^{0}y(s+\theta)d\zeta(\theta)-\frac{\sigma^2(s)y^{2\varrho}(s)}{2}\Big]ds+\int_0^t\sigma_1(s)y^{\varrho}(s)dB(s) \tag{2.2}$$
$$-t\int_{\mathbb{Y}}[\beta(u)-\ln(1+\beta(u))]\lambda(du)+\int_0^t\int_{\mathbb{Y}}\ln(1+\beta(u))\tilde{N}(ds,du).$$

On the other hand,

$$\int_0^t a_2(s)y(s-\tau)ds=\int_{-\tau}^{t-\tau}a_2(s+\tau)y(s)ds=\int_{-\tau}^{0}a_2(s+\tau)y(s)ds$$
$$+\int_0^{t-\tau}a_2(s+\tau)y(s)ds\le\int_{-\tau}^{0}a_2(s+\tau)y(s)ds+\int_0^{t}a_2(s+\tau)y(s)ds.$$

In other word, $\forall\ t \in \overline{R}_+$, we have

$$\ln z(t) - \ln z(0) = \int_0^t \Big[b(s) - \big(a_1(s) - a_2(s+\tau)\big)y(s) + a_3(s)\int_{-\infty}^0 y(s+\theta)d\zeta(\theta)$$
$$- \frac{\sigma^2(s)y^{2\varrho}(s)}{2}\Big]ds - t\int_{\mathbb{Y}}[\beta(u) - \ln(1+\beta(u))]\lambda(du) + \int_{-\tau}^0 a_2(s+\tau)y(s)ds$$
$$+ M_1(t) + M_2(t)$$

where $M_1(t) = \int_0^t \sigma_1(s)x^{\varrho}(s)dB(s)$ and $M_2(t) = \int_0^t \int_{\mathbb{Y}} \ln(1+\beta(u))\tilde{N}(ds,du)$. By (A1), we compute that

$$\int_0^t a_3(s)\int_{-\infty}^0 y(s+\theta)d\zeta(\theta)ds = \int_0^t a_3(s)\Big[\int_{-\infty}^{-s} y(s+\theta)d\zeta(\theta)ds + \int_{-s}^0 y(s+\theta)d\zeta(\theta)\Big]ds$$
$$= \int_0^t a_3(s)ds\int_{-\infty}^{-s} e^{\mathbf{r}(s+\theta)}y(s+\theta)e^{-\mathbf{r}(s+\theta)}d\zeta(\theta) + \int_{-t}^0 d\zeta(\theta)\int_{-\theta}^t a_3(s)y(s+\theta)ds$$
$$\leq a_3^u\|\xi\|_{c_g}\int_0^t e^{-\mathbf{r}s}ds\int_{-\infty}^0 e^{-\mathbf{r}\theta}d\zeta(\theta) + a_3^u\int_{-\infty}^0 d\zeta(\theta)\int_0^t y(s)ds$$
$$\leq a_3^u\|\xi\|_{c_g}\int_0^t e^{-\mathbf{r}s}ds\int_{-\infty}^0 e^{-2\mathbf{r}\theta}d\zeta(\theta) + a_3^u\int_{-\infty}^0 d\zeta(\theta)\int_0^t y(s)ds$$
$$\leq \frac{1}{\mathbf{r}}a_3^u\|\xi\|_{c_g}\mu_{\mathbf{r}}(1-e^{-\mathbf{r}t}) + a_3^u\int_0^t y(s)ds.$$

Consequently,

$$\int_{t-\tau}^t a_2(s+\tau)y(s)ds - \int_{-\tau}^0 a_2(s+\tau)y(s)ds + \ln z(t) - \ln z(0)$$
$$\leq \int_0^t \Big[b(s) - \big(a_1(s) - a_2(s+\tau) - a_3^u\big)y(s) - \frac{\sigma^2(s)y^{2\varrho}(s)}{2}\Big]ds \tag{2.3}$$
$$+ \frac{1}{\mathbf{r}}a_3^u\|\xi\|_{c_g}\mu_{\mathbf{r}}(1-e^{-\mathbf{r}t}) - t\int_{\mathbb{Y}}[\beta(u) - \ln(1+\beta(u))]\lambda(du) + M_1(t) + M_2(t),$$

where $M_1(t)$ is a real-valued continues local martingale vanishing at $t = 0$ and its quadratic form is given by $\langle M_1(t), M_1(t)\rangle = \int_0^t \sigma^2(s)y^{2\varrho}(s)ds$. By virtue of the exponential martingale inequality, for any positive constants T_0, α and β, we have

$$\mathcal{P}\Big\{\sup_{0\le t\le T_0}\Big[M_1(t)-\frac{\alpha}{2}\langle M_1(t),M_1(t)\rangle\Big]>\beta\Big\}\le e^{-\alpha\beta}.$$

Choose $T_0=k, \alpha=1, \beta=2\ln n$. Then it follows that

$$\mathcal{P}\Big\{\sup_{0\le t\le n}\Big[M_1(t)-\frac{1}{2}\langle M_1(t),M_1(t)\rangle\Big]>2\ln n\Big\}\le\frac{1}{k^2}.$$

By using the Borel-Cantelli lemma, for almost all $\omega\in\Omega$, there is a random integer $n_0=n_0(\omega)$ such that for $n\ge n_0$,

$$\sup_{0\le t\le n}\Big[M_1(t)-\frac{1}{2}\langle M_1(t),M_1(t)\rangle\Big]\le 2\ln n.$$

This is to say

$$M_1(t)\le 2\ln n+\frac{1}{2}\langle M_1(t),M_1(t)\rangle=2\ln n+\frac{1}{2}\int_0^t\sigma^2(s)y^{2\varrho}(s)ds$$

for all $0\le t\le n, n\ge n_0$ a.s. Substituting this inequality into (2.3), we can obtain that

$$\begin{aligned}&\int_{t-\tau}^{t}a_2(s+\tau)y(s)ds+\ln z(t)-\ln z(0)\\&\le\int_{-\tau}^{0}a_2(s+\tau)y(s)ds+\int_0^t\Big[b(s)-\Big(a_1(s)-a_2(s+\tau)-a_3^u\Big)y(s)\Big]ds\\&-\int_{\mathbb{Y}}[\beta(u)-\ln(1+\beta(u))]\lambda(du)+2\ln n+\frac{1}{\mathbf{r}}a_3^u\|\xi\|_{c_g}\mu_{\mathbf{r}}(1-e^{-\mathbf{r}t})+M_2(t)\end{aligned}\tag{2.4}$$

for all $0\le t\le n, n\ge n_0$ a.s. In addition, it follows from (2.4) that

$$\begin{aligned}&\sum_{0<t_k<t}\ln(1+J_k)+\ln z(t)-\ln z(0)\\&\le\sum_{0<t_k<t}\ln(1+J_k)+\int_{-\tau}^{0}a_2(s+\tau)y(s)ds\\&+\int_0^t\Big[b(s)-\Big(a_1(s)-a_2(s+\tau)-a_3^u\Big)y(s)\Big]ds+2\ln n+\frac{1}{\mathbf{r}}a_3^u\|\xi\|_{c_g}\mu_{\mathbf{r}}(1-e^{-\mathbf{r}t})\\&-t\int_{\mathbb{Y}}[\beta(u)-\ln(1+\beta(u))]\lambda(du)+M_2(t)\end{aligned}$$

for all $0 \le t \le n, n \ge n_0$ a.s. In other words, we have shown that

$$\begin{aligned}\ln y(t) - \ln y(0) \le & \sum_{0<t_k<t} \ln(1+J_k) + \int_{-\tau}^{0} a_2(s+\tau)y(s)ds \\ & + \int_{0}^{t} \Big[b(s) - \Big(a_1(s) - a_2(s+\tau) - a_3(s)\Big)y(s)\Big]ds + 2\ln n \\ & + \frac{1}{\mathbf{r}} a_3^u \|\xi\|_{c_g} \mu_{\mathbf{r}} (1 - e^{-\mathbf{r}t}) - t \int_{\mathbb{Y}} [\beta(u) - \ln(1+\beta(u))]\lambda(du) \\ & + M_2(t)\end{aligned} \tag{2.5}$$

for all $n-1 \le t \le n, n \ge n_0$ a.s. Therefore, $\forall\ t \in \overline{R}_+$, we get

$$\begin{aligned}\ln y(t) - \ln y(0) \le & \sum_{0<t_k<t} \ln(1+J_k) + \int_{-\tau}^{0} a_2(s+\tau)y(s)ds + \int_{0}^{t} b(s)ds + 2\ln n \\ & + \frac{1}{\mathbf{r}} a_3^u \|\xi\|_{c_g} \mu_{\mathbf{r}} (1 - e^{-\mathbf{r}t}) - t \int_{\mathbb{Y}} [\beta(u) - \ln(1+\beta(u))]\lambda(du) \\ & + M_2(t)\end{aligned}$$

for all $0 \le t \le n, n \ge n_0$ a.s. By (A3), one can get $\langle M_2 \rangle(t) = \int_0^t \int_{\mathbb{Y}} (\ln(1+\beta(u))^2 \lambda(du)ds \le c^2 t\lambda(\mathbb{Y})$, by the strong law of large numbers for local martingales(see, e.g., [43]), then we obtain

$$\lim_{t\to+\infty} \frac{M_2(t)}{t} = 0 \ \ a.s., \tag{2.6}$$

which is the required assertion by (A2). □

Theorem 2. *Suppose (A1)-(A4) hold, if* $G^* = 0$ *and* $\inf_{t\in\overline{R}_+}\{a_1(t) - a_2(t+\tau) - a_3^u\} > 0$*, then* $y(t)$ *modeled by* (1.4) *is non-persistent in the mean a.s.*

Proof. If $G^* = 0$, satisfying (2.5) and (A2), for arbitrarily $\varepsilon > 0$, there exists a constant T such that $t^{-1}\Big[\sum_{0<t_k<t} \ln(1+J_k) + \int_0^t b(s)ds\Big] + t^{-1} \int_{-\tau}^{0} a_2(s+\tau)y(s)ds + t^{-1}\frac{1}{\mathbf{r}} a_3^u \|\xi\|_{c_g} \mu_{\mathbf{r}}(1-e^{-\mathbf{r}t}) + \frac{2\ln k}{t} - \int_{\mathbb{Y}} [\beta(u) - \ln(1+\beta(u))]\lambda(du) + \frac{M_2(t)}{t} < \varepsilon$ for all $T \le k-1 \le t \le k,\ k \ge k_0$ a.s.

Substituting this inequality into (2.5) yields that

$$\begin{aligned}\ln y(t) - \ln y(0) \le & \sum_{0<t_k<t} \ln(1+J_k) + \int_{-\tau}^{0} a_2(s+\tau)y(s)ds + \int_{0}^{t} \Big[b(s) - \Big(a_1(s) \\ & - a_2(s+\tau) - a_3(s)\Big)y(s)\Big]ds \\ & + 2\ln k + \frac{1}{\mathbf{r}} a_3^u \|\xi\|_{c_g} \mu_{\mathbf{r}} (1 - e^{-\mathbf{r}t})\end{aligned}$$

$$
\begin{aligned}
&- \int_{\mathbb{Y}} [\beta(u) - \ln(1+\beta(u))]\lambda(du) + M_2(t)\\
&< \varepsilon t - \int_0^t \Big(a_1(s) - a_2(s+\tau) - a_3^u\Big) y(s)ds
\end{aligned}
$$

for all $T \le k-1 \le t \le k,\ k \ge k_0$ a.s.

Define $h(t) = \int_0^t y(s)ds$ and $I = \inf_{t\in\overline{R}_+} \Big[a_1(t) - a_2(t+\tau) - a_3^u\Big]$. The rest of the proof is similar to Theorem 3 in [13], so is omitted. □

Theorem 3. *Suppose (A1)-(A4) hold. If $G^* > 0$, then $y(t)$ of model* (1.4) *is weak persistence a.s.*

Proof. If the assertion is not true, let F be the set $F = \{\limsup_{t\to+\infty} y(t) = 0\}$, then $P(F) > 0$, on the basis of (2.2), we derive

$$
\begin{aligned}
&\sum_{0<t_k<t} \ln(1+J_k) + \ln z(t) - \ln z(0)\\
=& \sum_{0<t_k<t} \ln(1+J_k) + \int_0^t \Big[b(s) - a_1(s)y(s) + a_2(s)y(s-\tau)\\
&+ a_3(s)\int_{-\infty}^0 y(s+\theta)d\zeta(\theta) - \frac{\sigma^2(s)y^{2\varrho}(s)}{2}\Big]ds\\
&- t\int_{\mathbb{Y}} [\beta(u) - \ln(1+\beta(u))]\lambda(du) + M_1(t) + M_2(t),
\end{aligned}
$$

then divide by t, we get

$$
\begin{aligned}
&t^{-1}\ln y(t) - t^{-1}\ln y(0)\\
=& t^{-1}\sum_{0<t_k<t} \ln(1+J_k) + t^{-1}\int_0^t \Big[b(s) - a_1(s)y(s) + a_2(s)y(s-\tau)\\
&+ a_3(s)\int_{-\infty}^0 y(s+\theta)d\zeta(\theta) - \frac{\sigma^2(s)y^{2\varrho}(s)}{2}\Big]ds\\
&- \int_{\mathbb{Y}} [\beta(u) - \ln(1+\beta(u))]\lambda(du) + \frac{M_1(t)}{t} + \frac{M_2(t)}{t}.
\end{aligned}
\tag{2.7}
$$

On the other hand, for $\forall\ \omega \in F$, we have $\lim_{t\to+\infty} x(t,\omega) = 0$. Consequently, by the law of large numbers for local martingales, we obtain that $\lim_{t\to+\infty} M_i(t)/t = 0, i = 1,2$. Substituting this equality into (2.7), one can get a contradiction

$$
0 \ge \limsup_{t\to+\infty}[t^{-1}\ln y(t,\ \omega)] = G^* + \limsup_{t\to+\infty} t^{-1}\int_0^t a_2(s)y(s-\tau)ds
$$

$$+\limsup_{t\to+\infty} t^{-1}\int_0^t a_3(s)\int_{-\infty}^0 y(s+\theta)d\zeta(\theta)ds \geq G^* > 0.$$

□

Remark 1. *Based on Theorems 1-3, we can point out that G^* is the threshold between weak persistence and extinction for the population $y(t)$ by (A1)-(A4) if $\inf_{t\in\bar{R}_+}\left\{a_1(t)-a_2(t+\tau)-a_3^u\right\}\geq 0$ holds.*

Lastly we show that $y(t)$ modeled by (1.4) is stochastic permanence in some cases. We also need the assumption as follows:

(A5): There exist two positive constants m and M such that $m \leq \prod_{0<t_k<t}(1+J_k) \leq M$ for all $t > 0$.

Theorem 4. *Suppose (A1)-(A5) hold, if $b_* - \int_{\mathbb{Y}} \frac{\beta^2(u)}{1+\beta(u)}\lambda(du) > 0$, $\varrho < 1$, and $a_2(t), a_3(t) \geq 0$, then $y(t)$ represented by* (1.4) *will be stochastic permanence.*

Proof. First, we prove that for arbitrary $\varepsilon > 0$, there exists a constant $\vartheta_1 > 0$ such that $\liminf_{t\to+\infty}\mathcal{P}\{y(t)\leq \vartheta_1\} \geq 1-\varepsilon$.

Let $0.5 < p = 2\varrho - 1 < 1$ and choose $\rho_1 \in (0, 2\mathbf{r})$, we obtain

$$\begin{aligned}
dz^p(t) =& pz^{p-1}(t)dz(t) + \frac{1}{2}p(p-1)z^{p-2}(t)(dz(t))^2\\
=& pz^{p-1}(t)\Big[\Big(z(t)\Big(b(t) - \prod_{0<t_k<t}(1+J_k)a_1(t)z(t)\\
&+ \prod_{0<t_k<t-\tau}(1+J_k)a_2(t)z(t-\tau)\\
&+ a_3(t)\int_{-\infty}^{0}\prod_{0<t_k<t+\theta}(1+J_k)z(t+\theta)d\zeta(\theta)\Big)\Big)dt\\
&+\Big(\prod_{0<t_k<t}(1+J_k)\Big)^{\varrho}\sigma_1(t)z^{1+\varrho}(t)dB(t)\Big]\\
&+\frac{1}{2}p(p-1)\Big(\prod_{0<t_k<t}(1+J_k)\Big)^{2\varrho}\sigma^2(t)z^{p+2\varrho}(t)dt\\
&+\int_{\mathbb{Y}}[(1+\beta(u))^p - 1 - p\beta(u)]\lambda(du)z^p(t)\\
&+\int_{\mathbb{Y}}[(1+\beta(u))^p - 1]\tilde{N}(dt,du)z^p(t)
\end{aligned}$$

$$\begin{aligned}
\leq& pz^{p-1}(t)\Big[\Big(z(t)\Big(b(t) - ma_1(t)z(t) + Ma_2(t)z(t-\tau)\\
&+ Ma_3(t)\int_{-\infty}^{0}z(t+\theta)d\zeta(\theta)\Big)\Big)dt + \Big(\prod_{0<t_k<t}(1+J_k)\Big)^{\varrho}\sigma_1(t)z^{1+\varrho}(t)dB(t)\Big]
\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{2}p(p-1)\Big(\prod_{0<t_k<t}(1+J_k)\Big)^{2\varrho}\sigma^2(t)z^{p+2\varrho}(t)dt\\
&+\int_{\mathbb{Y}}[(1+\beta(u))^p-1-p\beta(u)]\lambda(du)z^p(t)\\
&+\int_{\mathbb{Y}}[(1+\beta(u))^p-1]\tilde{N}(dt,du)z^p(t)\\
\le&\Big[b(t)pz^p(t)+\frac{p^2M^2a_2^2(t)z^{2p}(t)}{4}+z^2(t-\tau)+\frac{p^2M^2a_3^2(t)z^{2p}(t)}{4}\\
&+\int_{-\infty}^{0}z^2(t+\theta)d\zeta(\theta)\Big]dt+\Big(\prod_{0<t_k<t}(1+J_k)\Big)^{\varrho}p\sigma_1(t)z^{p+\varrho}(t)dB(t)\\
&-\frac{1}{2}p(1-p)m^{2\varrho}\sigma^2(t)z^{p+2\varrho}(t)dt+\int_{\mathbb{Y}}[(1+\beta(u))^p-1-p\beta(u)]\lambda(du)z^p(t)\\
&+\int_{\mathbb{Y}}[(1+\beta(u))^p-1]\tilde{N}(dt,du)z^p(t)\\
=&F(z(t))dt-\Big[\rho_1y^p(t)+e^{\rho_1\tau}z^2(t)-z^2(t-\tau)\\
&-\int_{-\infty}^{0}z^2(t+\theta)d\zeta(\theta)+\mu_{\mathbf{r}}z^2(t)\Big]dt+\Big(\prod_{0<t_k<t}(1+J_k)\Big)^{\varrho}p\sigma_1(t)z^{p+\varrho}(t)dB(t)\\
&+\int_{\mathbb{Y}}[(1+\beta(u))^p-1-p\beta(u)]\lambda(du)z^p(t)\\
&+\int_{\mathbb{Y}}[(1+\beta(u))^p-1]\tilde{N}(dt,du)z^p(t)
\end{aligned}
$$

where

$$
\begin{aligned}
F(z)=&e^{\rho_1\tau}z^2+\mu_{\mathbf{r}}z^2+\Big(\rho_1+b(t)p\Big)z^p+p^2a_2^2(t)z^{2p}+p^2a_3^2(t)z^{2p}\\
&-\frac{1}{2}p(1-p)m^{2\varrho}\sigma^2(t)z^{p+2\varrho}+\int_{\mathbb{Y}}[(1+\beta(u))^p-1-p\beta(u)]\lambda(du)z^p.
\end{aligned}
$$

According to $p>0$, (A2) and (A4), we have $F(z)$ is bounded in R_+, namely

$$
K=\sup_{z\in R_+}F(z)<+\infty.
$$

Thus we show

$$
\begin{aligned}
dz^p(t)=&[K-\rho_1z^p(t)-e^{\rho_1\tau}z^2(t)+z^2(t-\tau)]dt+\int_{-\infty}^{0}z^2(t+\theta)d\zeta(\theta)dt\\
&-\mu_{\mathbf{r}}z^2(t)dt+\Big(\prod_{0<t_k<t}(1+J_k)\Big)^{\varrho}p\sigma_1(t)z^{p+\varrho}(t)dB(t)\\
&+\int_{\mathbb{Y}}[(1+\beta(u))^p-1]\tilde{N}(dt,du)z^p(t).
\end{aligned}
$$

Applying Itô's formula leads to

$$d[e^{\rho_1 t}z^p(t)] =e^{\rho_1 t}[\rho_1 z^p(t)dt + dz^p(t)] \le e^{\rho_1 t}\Big[K - e^{\rho_1\tau}z^2(t) + z^2(t-\tau) + \int_{-\infty}^{0} z^2(t+\theta)d\zeta(\theta) - \mu_{\mathbf{r}} z^2(t)\Big]dt + e^{\rho_1 t}\Big(\prod_{0<t_k<t}(1+J_k)\Big)^{\varrho} p\sigma_1(t)z^{p+\varrho}(t)dB(t).$$

Hence we derive that

$$\begin{aligned}
e^{\rho_1 t}E[z^p(t)] \le& \xi^p(0) + \frac{e^{\rho_1 t}K}{\rho_1} - \frac{K}{\rho_1} - E\int_0^t e^{\rho_1 s+\rho_1\tau}z^2(s)ds + E\int_0^t e^{\rho_1 s}z^2(s-\tau)ds \\
&+ E\int_0^t e^{\rho_1 s}\int_{-\infty}^0 z^2(s+\theta)d\zeta(\theta)ds - E\int_0^t \mu_{\mathbf{r}} e^{\rho_1 s}z^2(s)ds \\
=& \xi^p(0) + \frac{e^{\rho_1 t}K}{\rho_1} - \frac{K}{\rho_1} - E\int_0^t e^{\rho_1 s+\rho_1\tau}z^2(s)ds + E\int_{-\tau}^{t-\tau} e^{\rho_1 s+\rho_1\tau}z^2(s)ds \\
&+ E\int_0^t e^{\rho_1 s}\int_{-\infty}^0 z^2(s+\theta)d\zeta(\theta)ds - E\int_0^t \mu_{\mathbf{r}} e^{\rho_1 s}z^2(s)ds \\
\le& \xi^p(0) + \frac{e^{\rho_1 t}K}{\rho_1} - \frac{K}{\rho_1} + \int_{-\tau}^0 e^{\rho_1 s+\rho_1\tau}z^2(s)ds \\
&+ E\int_0^t e^{\rho_1 s}\int_{-\infty}^0 z^2(s+\theta)d\zeta(\theta)ds - E\mu_{\mathbf{r}}\int_0^t e^{\rho_1 s}z^2(s)ds.
\end{aligned}$$

By (A4), we have

$$\begin{aligned}
&\int_0^t e^{\rho_1 s}\int_{-\infty}^0 z^2(s+\theta)d\zeta(\theta)ds = \int_0^t e^{\rho_1 s}\Big[\int_{-\infty}^{-s} z^2(s+\theta)d\zeta(\theta) + \int_{-s}^0 z^2(s+\theta)d\zeta(\theta)\Big]ds \\
&= \int_0^t e^{\rho_1 s}ds\int_{-\infty}^{-s} e^{2\mathbf{r}(s+\theta)}z^2(s+\theta)e^{-2\mathbf{r}(s+\theta)}d\zeta(\theta) + \int_{-t}^0 d\zeta(\theta)\int_{-\theta}^t e^{\rho_1(s)}z^2(s+\theta)ds \\
&= \int_0^t e^{\rho_1 s}ds\int_{-\infty}^{-s} e^{2\mathbf{r}(s+\theta)}z^2(s+\theta)e^{-2\mathbf{r}(s+\theta)}d\zeta(\theta) + \int_{-t}^0 d\zeta(\theta)\int_0^{t+\theta} e^{\rho_1(s-\theta)}z^2(s)ds \\
&\le \|\xi\|_{c_g}^2\int_0^t e^{(\rho_1-2\mathbf{r})s}ds\int_{-\infty}^0 e^{-2\mathbf{r}\theta}d\zeta(\theta) + \int_{-\infty}^0 e^{-\rho_1\theta}d\zeta(\theta)\int_0^t e^{\rho_1 s}z^2(s)ds \\
&\le \|\xi\|_{c_g}^2\mu_{\mathbf{r}} t + \mu_{\mathbf{r}}\int_0^t e^{\rho_1 s}z^2(s)ds.
\end{aligned}$$

This immediately implies that

$$\limsup_{t\to+\infty} E[z^p(t)] \le \frac{K}{\rho_1} = H. \tag{2.8}$$

Consequently,

$$\limsup_{t\to+\infty} E(y^p(t)) = \limsup_{t\to+\infty}\Big[\prod_{0<t_k<t}(1+J_k)\Big]^p E(z^p(t)) \le \Big[M^p\frac{K}{\rho_1}\Big] = M_1.$$

Thus for any given $\varepsilon > 0$, let $\vartheta_1 = M_1^{\frac{1}{p}}/\varepsilon^{\frac{1}{p}}$, by virtue of Chebyshev's inequality, we can derive that

$$\mathcal{P}\{y(t) > \vartheta_1\} = \mathcal{P}\{y^p(t) > \vartheta_1^p\} \le E[y^p(t)]/\vartheta_1^p.$$

That is to say $\limsup_{t\to+\infty}\mathcal{P}\{y(t) > \vartheta_1\} \le \varepsilon$. Consequently, $\liminf_{t\to+\infty}\mathcal{P}\{y(t) \le \vartheta_1\} \ge 1-\varepsilon$.

Next, we claim that for arbitrary $\varepsilon > 0$, there exists a constant $\vartheta_2 > 0$ such that $\liminf_{t\to+\infty}\mathcal{P}\{y(t) \ge \vartheta_2\} \ge 1-\varepsilon$. One can obtain that

$$\begin{aligned} d\Big(\frac{1}{z(t)}\Big) =&\Big[-\frac{b(t)-\int_{\mathbb{Y}}\frac{\beta^2(u)}{1+\beta(u)}\lambda(du)}{z(t)} - \frac{a_2(t)z(t-\tau)}{z(t)} \\ &-\frac{a_3(t)\int_{-\infty}^{0}\prod_{0<t_k<t+\theta}(1+J_k)z(t+\theta)d\zeta(\theta)}{z(t)} + a_1(t) + \sigma^2(t)\Big(\prod_{0<t_k<t}(1+J_k)\Big)^{2\varrho} \\ &\times z^{2\varrho-1}(t)\Big]dt - \sigma_1(t)\Big(\prod_{0<t_k<s}(1+J_k)\Big)^{\varrho} z^{\varrho-1}(t)dB(t) + \frac{1}{z(t)}\int_{\mathbb{Y}}\Big[\frac{1}{(1+\beta(u))} \\ &-1\Big]\tilde{N}(dt,du). \end{aligned} \tag{2.9}$$

Integrating from 0 to t and taking expectations on the both sides of (2.9) we get

$$\begin{aligned} E\Big[\frac{1}{z(t)}\Big] =&E\Big[\frac{1}{z(0)}\Big] + \int_0^t\Big(E\Big[\frac{-\big(b(s)-\int_{\mathbb{Y}}\frac{\beta^2(u)}{1+\beta(u)}\lambda(du)\big)}{z(s)}\Big] + a_1(s) \\ &+\sigma^2(s)\Big(\prod_{0<t_k<t}(1+J_k)\Big)^{2\varrho}E[z^{2\varrho-1}(s)] - a_2(s)E\Big[\frac{z(s-\tau)}{z(s)}\Big] \\ &-a_3(s)E\Big[\frac{\int_{-\infty}^{0}\prod_{0<t_k<t+\theta}(1+J_k)z(s+\theta)d\zeta(\theta)}{z(s)}\Big]\Big)ds. \end{aligned}$$

Hence

$$\begin{aligned} \frac{dE[1/z(t)]}{dt} =& -\Big(b(t)-\int_{\mathbb{Y}}\frac{\beta^2(u)}{1+\beta(u)}\lambda(du)\Big)E\big[1/z(t)\big] + a_1(t) \\ &+\sigma^2(t)\Big(\prod_{0<t_k<t}(1+J_k)\Big)^{2\varrho}E[z^{2\varrho-1}(t)] - a_2(t)E\Big[\frac{z(t-\tau(t))}{z(t)}\Big] \\ &-a_3(t)E\Big[\frac{\int_{-\infty}^{0}\prod_{0<t_k<t+\theta}z(t+\theta)d\zeta(\theta)}{z(t)}\Big]. \end{aligned} \tag{2.10}$$

Considering the equation

$$\frac{dE[1/z_1(t)]}{dt} = -\Big(b(t) - \int_{\mathbb{Y}} \frac{\beta^2(u)}{1+\beta(u)}\lambda(du)\Big)E\Big[1/z_1(t)\Big] + a_1(t) + \sigma^2(t)M^{2\varrho}H. \tag{2.11}$$

For $\forall\varepsilon > 0$, there exists $T_1 > 0$, such that $b(t) > b_* - \varepsilon$ for all $t > T_1$. Using the same method (Theorem 4.5 in [44]), we have

$$\lim_{t\to+\infty} E\Big[\frac{1}{z_1(t)}\Big] \le d,$$

where

$$d = \frac{2(\sigma^2)^u M^{2\varrho}H}{b_* - \int_{\mathbb{Y}} \frac{\beta^2(u)}{1+\beta(u)}\lambda(du)} + \frac{\exp\{-\int_0^T \Big(b(s) - \int_{\mathbb{Y}} \frac{\beta^2(u)}{1+\beta(u)}\lambda(du)\Big)ds\}}{z_1(0)}.$$

Taking into consideration (2.8), there exists $T > T_1 > 0$ such that $E[z^{2\varrho-1}(t)] \le H$ for all $t > T$. Thus from (2.10) and (2.11), using the comparison theorem for ODEs yields that

$$E\Big[\frac{1}{z(t)}\Big] \le E\Big[\frac{1}{z_1(t)}\Big]$$

for $t > T$, which implies that

$$\limsup_{t\to+\infty} E\Big[\frac{1}{z(t)}\Big] \le \limsup_{t\to+\infty} E\Big[\frac{1}{z_1(t)}\Big] = \lim_{t\to+\infty} E\Big[\frac{1}{z_1(t)}\Big] \le d.$$

Thus

$$\limsup_{t\to+\infty} E\Big[\frac{1}{y(t)}\Big] = \lim_{t\to+\infty} E\Big[\frac{1}{\prod_{0<t_k<t}(1+J_k)z(t)}\Big] = m^{-1}d.$$

So for any given $\varepsilon > 0$, let $\vartheta_2 = \frac{m\varepsilon}{d}$, then the desired assertion follows from the Chebyshev inequality. □

Remark 2. *It is easy to see that the Theorem 1-3 coincide with the Theorem 2-4 in Liu et.al [13] when* $a_2(t) = a_3(t) \equiv 0, J_k = 0, \beta(u) = 0$ *and* $\varrho = 1$ *in model* (1.4), *respectively.*

Remark 3. *Taking notice of* G^* *in Theorem 1-3, we can find that the impulsive perturbation does not affect extinction, non-persistent in the mean and weak persistence satisfying the conditions that impulsive perturbations exist upper bound.*

Remark 4. *Unlike earlier studies, we do not employ Lyapunov methods in establishing the sufficient conditions of stochastic permanence. As we know, in general, it is not easy to construct suitable Lyapunov function to deal with stochastic permanence for stochastic differential equation with infinite delay. Therefore, we use the comparison theorem of ordinary differential equation to derive the sufficient condition of stochastic permanence.*

Remark 5. *Paying attention to* G^* *in Theorem 1-3, where* $\int_{\mathbb{Y}} [\beta(u) - \ln(1+\beta(u))]\lambda(du) > 0$*(see Lemma 1.2 in Ref. [17]), we obtain the result that the jump process may exert a considerable negative impact on the population and can lead to the extinction (see case 1 in section 4), which accords with biological significance.*

3. Global asymptotic stability

In this section, we will gain sufficient criteria of the global asymptotic stability for model (1.4) without Lévy jumps and impulsive perturbation:

$$\begin{cases} dy(t) = y(t)\Big[b - a_1 y(t) + a_2 y(t-\tau) + a_3 \int_{-\infty}^{0} y(t+\theta)d\zeta(\theta)\Big]dt \\ \qquad + \sigma_1(t) y^{1+\varrho}(t) dB(t) \end{cases} \tag{3.1}$$

with the same initial value of model (1.4).

Theorem 5. *Suppose (A2)-(A4) hold. If* $a_2 \le 0, a_3 \le 0, a_1 + a_2 + a_3 > 0$*, then the positive solution of model* (3.1) *is globally asymptotically stable.*

Proof. Let $y(t)$ and $y^*(t)$ be two arbitrary solutions of model (1.4). Define

$$V(t) = |\ln y(t) - \ln y^*(t)| - a_2 \int_{t-\tau}^{t} |y(s) - y^*(s)|ds - a_3 \int_{-\infty}^{0}\int_{t+\theta}^{t} |y(s) - y^*(s)|ds d\zeta(\theta).$$

A calculation of the right differential $D^+V(t)$, and then making use of the generalized Itô's formula, we can observe

$$\begin{aligned} D^+V(t) =& \mathrm{sgn}(y(t) - y^*(t))\Big(- a_1(y(t) - y^*(t)) + a_2(y(t-\tau) - y^*(t-\tau)) \\ & + a_3\Big(\int_{-\infty}^{0} \Big(y(t+\theta) - y^*(t+\theta)\Big)d\zeta(\theta) - \frac{\sigma^2}{2}(u^{2\nu}(t) - (u^*(t))^{2\nu})\Big)dt \\ & - a_2|y(t) - y^*(t)|dt + a_2|y(t-\tau) - y^*(t-\tau)|dt \\ & - a_3 \int_{-\infty}^{0} |y(t) - y^*(t)|d\nu(\theta)dt + a_3 \int_{-\infty}^{0} |y(t+\theta) - y^*(t+\theta)|d\nu(\theta)dt \\ =& - a_1|y(t) - y^*(t)|dt - a_2|y(t-\tau) - y^*(t-\tau)|dt \\ & - a_3\Big(\int_{-\infty}^{0} |y(t+\theta) - y(t+\theta)|d\nu(\theta)\Big)dt - \frac{\sigma^2(t)}{2}|y^{2\nu}(t) - (y^*(t))^{2\nu}|dt \\ & - a_2|y(t) - y^*(t)|dt + a_2|y(t-\tau) - y^*(t-\tau)|dt - a_3|y(t) - y^*(t)|dt \\ & + a_3 \int_{-\infty}^{0} |y(t+\theta) - y^*(t+\theta)|d\nu(\theta)dt \\ =& - (a_1 + a_2 + a_3)|y(t) - y^*(t)|dt - \frac{\sigma^2(t)}{2}|y^{2\nu}(t) - (y^*(t))^{2\nu}|dt. \end{aligned}$$

Integrating both sides and then taking the expectation yields

$$V(t) \le V(0) - \int_{0}^{t} (a_1 + a_2 + a_3)|y(s) - y^*(s)|ds.$$

In other words, we have already shown that

$$V(t) + \int_0^t (a_1 + a_2 + a_3)|y(s) - y^*(s)|ds \leq V(0) < \infty.$$

Because of $a_1 + a_2 + a_3 > 0$, we derive

$$|y(t) - y^*(t)| \in L^1[0, +\infty).$$

Similar to Theorem 11 in [26] and Theorem 3.7 in [45], we have

$$\lim_{t\to+\infty} |y(t) - y^*(t)| = 0.$$

This completes the proof of Theorem 5. □

4. Numerical simulations

In this section, we present an example to illustrate the results. Here let the probability measure $\zeta(\theta)$ be e^θ on $(-\infty, 0]$. Hence the model (1.4) will be rewritten as

$$\begin{cases} dy(t) = y(t)\Big[b(t) - a_1(t)y(t) + a_2(t)y(t-\tau) + a_3(t)e^{-t}\displaystyle\int_{-\infty}^{0} e^\theta \xi(\theta)d\theta \\ \qquad + a_3(t)e^{-t}\displaystyle\int_0^t e^\theta y(\theta)d\theta\Big]dt + \sigma_1(t)y^{1.8}(t)dB(t) \\ \qquad + y(t^-)\displaystyle\int_{\mathbb{Y}} \beta(u)\tilde{N}(dt, du), \\ \qquad t \neq t_k, \quad K \in N, \\ y(t_k^+) - y(t_k) = J_k y(t_k), \quad k \in N. \end{cases} \tag{4.1}$$

Using the Euler scheme to discretize this equation [46], choosing $\xi(\theta) = e^{-0.5\theta}$ and $\tau \equiv 0.3$, we can obtain the discrete approximate solution of (3.1):

$$\begin{cases} y_{k+1} = y_k + y_k\Big[r(k\Delta t) - a(k\Delta t)y_k + b(k\Delta t)y_{k-300} + c(k\Delta t)e^{-k\Delta t}\displaystyle\int_{-\infty}^{0} e^{0.5\theta}d\theta \\ \qquad + c(k\Delta t)e^{-k\Delta t}\displaystyle\sum_{j=0}^{k} \omega_j^{(k)} e^{j\Delta t} y_j\Big]\Delta t + \sigma(k\Delta t)y_k^{1.8}\Delta B_k + y_k\beta(u)\Delta\tilde{N}_k, \\ \qquad t \neq t_k, \quad K \in N, \\ y_{k+1} - y_k = J_k y_k, \quad t = t_k, \quad k \in N, \end{cases} \tag{4.2}$$

where $\Delta B_k = B((k+1)\Delta t) - B(k\Delta t)$, $\Delta\tilde{N}_k = \tilde{N}((k+1)\Delta t) - \tilde{N}(k\Delta t)$, $k = 0, 1, 2, \cdots$. The general composite θ-rule has weights

$$\{\omega_0^{(k)}, \omega_1^{(k)}, \cdots, \omega_k^{(k)}\} = \{\theta, 1, \cdots, 1 - \theta\}, \ \theta \in [0, 1]$$

and $\sum_{j=0}^{k} \omega_j^{(k)} = k, \ k \geq 0.$

Here, we choose $b(t) = 0.25 + 0.02 \cos t, a_1(t) = 0.24 + 0.01 \cos t, a_2(t) = 0.04, a_3(t) = 0.08, \sigma_1(t) = 0.02, \beta(u) = 0.98, \varrho = -0.2, t_k = 10k$ and step size $\Delta t = 0.001$. The only difference between conditions of Figure 1(A)-(C) is that the representation of J_k is different.

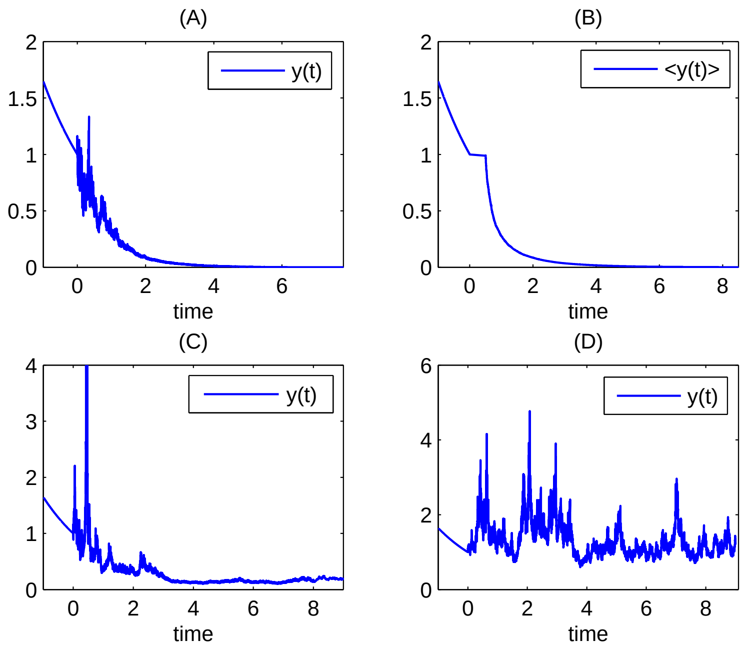

Figure 1. The horizontal axis in this and following figures represent the time t. (A) is with $J_k = 0$; (B)is with $J_k = e^{0.5} - 1$; (C)is with $J_k = e^{0.7} - 1$; (D) is with $J_k = e^{\frac{(-1)^{k+1}}{k}} - 1$.

Case 1. If $J_k = 0$ and the conditions of Theorem 1 have been satisfied, then the population $y(t)$ will be extinct(see Figure 1(A)).

Case 2. Considering $J_k = e^{0.5} - 1$, and the conditions of Theorem 2 hold, then the population $y(t)$ will be non-persistent in the mean(see Figure 1(B)).

Case 3. If $J_k = e^{0.7} - 1$, and the conditions of Theorem 3 are fulfilled, then the population $y(t)$ will be weak persistence(see Figure 1(C)).

Case 4. Considering $J_k = e^{\frac{(-1)^{k+1}}{k}} - 1$, and the conditions of Theorem 4 hold, then the population $y(t)$ will be stochastic permanence(see Figure 1(D)). By comparing with Figure 1(A)–(C), we can find that the impulsive perturbations have ability to transform the properties of the model.

Generally speaking, the above results show that the impulse perturbations which are bounded never influence the persistence and extinction. However, if the impulsive perturbations are unbounded, the persistence and extinction are severely affected. Usually, we think that making great efforts to continuous exploitation and harvesting may be described by unbounded impulsive perturbations.

5. Conclusion

In this paper, we study the basic features of a stochastic logistic model with infinite delay in presence of Lévy jumps and impulsive perturbations to understand the dynamics in a complicated environment. We have established sufficient conditions for the existence of global positive solution and obtained sufficient conditions for extinction, non-persistence in the mean, weak persistence, stochastic permanence and global asymptotic stability of model. Moreover, our investigation also reveals that impulsive perturbations which may represent natural and human factors play an important role in protecting the population, even if population suffers sudden environmental changes described by Lévy jumps, such as earthquakes, hurricanes, epidemics, etc.

Finally, we would like to talk about some interesting topics deserving further discussion. On the one hand, one may propose the realistic but complex model, such as considering multi-dimensional system. The motivation of investigating this is that biological systems are composed of multiple populations, such as prey, predator, etc. On the other hand, it is also significant to incorporate the telegraph noise, which can be modeled by a continuous-time Markov chain [23], into model (1.4). We will look into these topics in following work.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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